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CROP PRODUCTION. VOLUME 1: MODEL STRUCTURE
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A DISTRIBUTION BENEFITS MODEL
FOR IMPROVED INFORMATION
ON WORLDWIDE CROP PRODUCTION
VOLUME I
MODEL STRUCTURE AND
APPLICATION TO WHEAT



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FINAL

A DISTRIBUTION BENEFITS MODEL
FOR IMPROVED INFORMATION
ON WORLDWIDE CROP PRODUCTION
VOLUME I
MODEL STRUCTURE AND
APPLICATION TO WHEAT

Prepared for
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NOTE OF TRANSMITTAL

This report is prepared for the National Aeronautics and Space Administration, Office of Applications, under Contract NASW-2558.

The methods developed in this report for estimating benefits of improved information are the best ECON is aware of at the time of writing. However, the subject is immensely complex, and it is possible that later work will improve on the methods and the accuracy of the results. ECON has maintained a conservative viewpoint on potential economic benefits of improved information, so that the estimates presented are more likely to be on the low side than the high side.

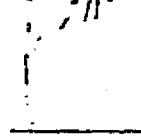
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Submitted by:



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ABSTRACT

Extensions and refinements of ECON's previously developed models for distribution benefits of improved information on agricultural production permit consideration of international trade and an unlimited decision-making horizon. The improved model is suitable for the study of benefits of worldwide information on a variety of crops. Application to the previously studied case of worldwide wheat production shows that about \$108 million per year of distribution benefits to the United States would be achieved by a satellite-based wheat information system meeting the goals of LACIE. The model also indicates that improved information alone will not change world stock levels unless production itself is stabilized. The United States benefits mentioned above are associated with the reduction of price fluctuations within the year and the more effective use of international trade to balance supply and demand. Price fluctuations from year to year would be reduced only if production variability were itself reduced.

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1. INTRODUCTION

Previous ECON reports [1, 2, 3, 5] have presented models of information, trade, and consumption in relation to wheat markets and have given estimates of potential benefits due to improved (LANDSAT) information on worldwide and domestic wheat production. Extensions and refinements of the earlier models have been developed in order to support distribution benefits studies for improved public information on other crops than wheat. In addition, the improved models provide some verification and clarification of previously published benefits estimates for the case of wheat.

In "U.S. Benefits of Worldwide Wheat Crop Information from a LANDSAT System" [5], completed in August, 1975 and revised January 31, 1976, ECON reported potential benefits to the United States of LANDSAT information on worldwide wheat production of about \$174 million annually. These benefits included both distribution effects and production effects of improved information. Also in 1975, ECON completed an investigation [3] of distribution benefits alone, suggesting that these might amount to a small fraction of the above \$174 million.

The present report describes an extended distribution model for storable commodities and illustrates

its application with a calculation of distribution benefits to the United States associated with improved wheat production forecasts on the rest of the world. This calculation verifies that about \$108 million per year of benefits to the United States would be achieved by an information system meeting the goals of LACIE. Later reports will present the results of applying this model to other small grains, soybeans, corn, sugar, and potatoes. The present model is nonlinear, dynamic, and decision oriented. It assumes that trade and consumption decisions are made in a free market, making optimal use of all available information, and planning with an unlimited horizon. The "decision policies" resulting from these assumptions are calculated by a dynamic programming technique. After the "decision policies" are determined, monte carlo simulation is used to determine average annual values, benefits, and other interesting properties of the system.

The methods discussed in this study have been developed primarily in order to evaluate potential improvements in information which are expected from LANDSAT and its successors. However, the results are equally valid if applied to improved information from any source. Further, the models can be applied to the evaluation of other changes in the conditions of world commodity markets

than information. For example, one could determine the economic consequences of particular regulatory policies on commodity markets, or the economic consequences of the formation and use of a world food fund under specified operating policies.

2. ECONOMICS OF CROP DISTRIBUTION

This study is largely concerned with the fact that the economic value of a fixed supply of a crop is significantly affected by the patterns of distribution. For example, if the world production of wheat in 1975-6 is 350 million metric tons and if 300 million metric tons are consumed in the first 6 months of the crop year, while 50 million metric tons are consumed in the next 6 months, the economic welfare of the world's wheat consumers is less than it would be under a more nearly uniform consumption pattern. This is a temporal distribution effect. Similarly, different spatial distribution patterns lead to different economic welfare results. If 40 million metric tons of the 350 million metric tons of world production were consumed in the United States, consumers in both the United States and in the rest of the world would be economically less well off than if the United States consumption were about 20 million metric tons, the quantity demanded in the United States at current commodity market prices.

The quantification of such economic welfare differences is the first step in determining the economic value of information that is expected to affect the distribution of a crop. The information of interest here is any estimate of production in the United States or in other

countries, provided it is available early enough to influence trade or storage decisions affecting some part of the annual crop. The value of such information is realized through the trade and storage decisions that it influences. The quantification of the value associated with a given distribution pattern is based on the concept of demand function.^{*} In a given region during a given time period, for example, in the United States the first quarter of 1976, it is assumed that the total consumption of wheat is a definite function of the average price at which it is offered. By considering the implications of this relationship, we can assign a meaningful economic value to the wheat consumed during the period. In Figure 2.3, showing a linear estimate of a demand function, the shaded area represents the gross value to the consumers associated with the consumption level y . In this situation, consumers pay a price p for each unit consumed, but the economic value of the consumed crop is not simply py . Without knowledge of the demand functions, we could only say that the economic value is not less than py , since consumers are demonstrating their willingness to pay that much. Having the entire demand function, we can determine how much consumers would have been willing to pay (if necessary) for the first unit, the second, etc., up to

^{*} A discussion of the validity of this approach is given in Appendix B.

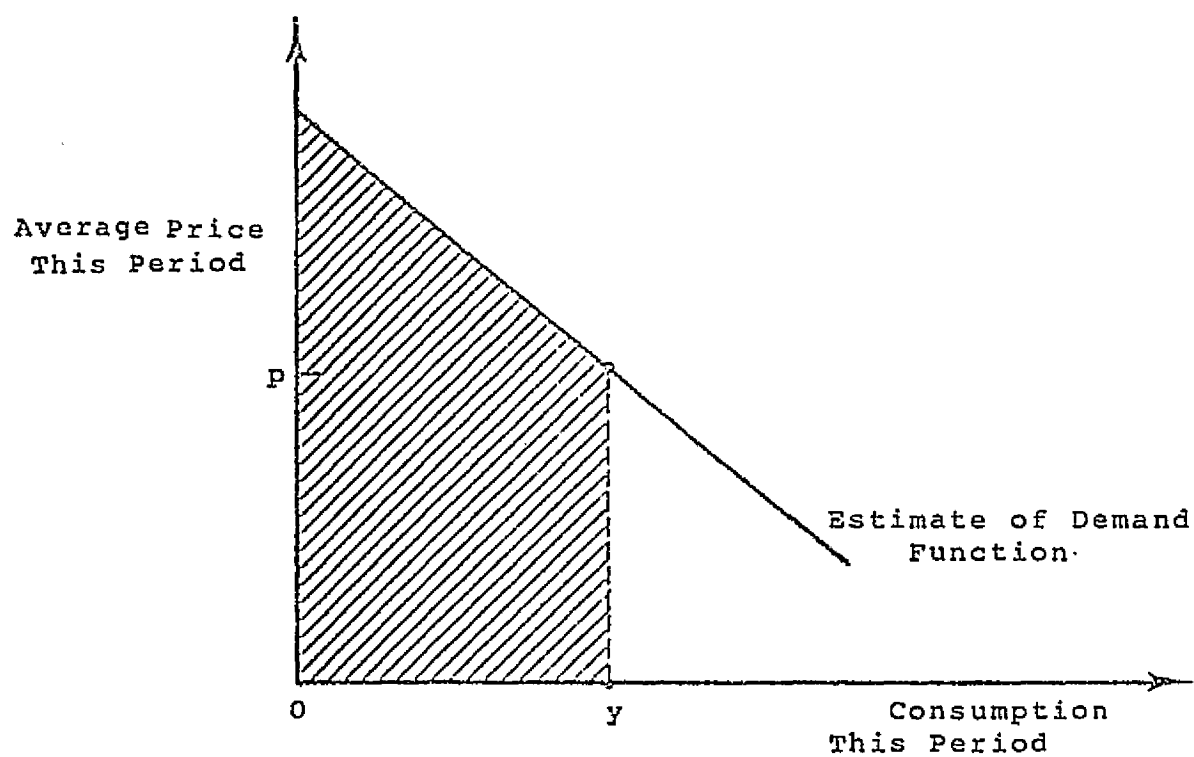


Figure 2.3 Demand Function and Gross
Economic Value of Consumption

the actual amount consumed y . The sum of these amounts of money is the integral under the graph of the demand function, or the shaded area in Figure 2.3. This is the true economic value of the consumption during the period.

Now suppose that the same demand function applies during two consecutive periods, and suppose its equation is:

$$p = b - ay ,$$

where a and b are positive. Now suppose a total quantity Y is available for consumption over the two periods. If y_1 is consumed during the first period and $Y - y_1$ in the second, the total economic value in the two periods is:

$$\begin{aligned} by_1 - \frac{1}{2} a y_1^2 + b (Y - y_1) - \frac{1}{2} a (Y - y_1)^2 \\ = - a y_1^2 + a Y y_1 + b Y - \frac{1}{2} a Y^2 . \end{aligned}$$

This quadratic function is graphed in Figure 2.4. Its greatest value is achieved when $y_1 = \frac{1}{2} Y$ the case of equal consumption in the two periods. The loss to the consumers associated with any other consumption pattern

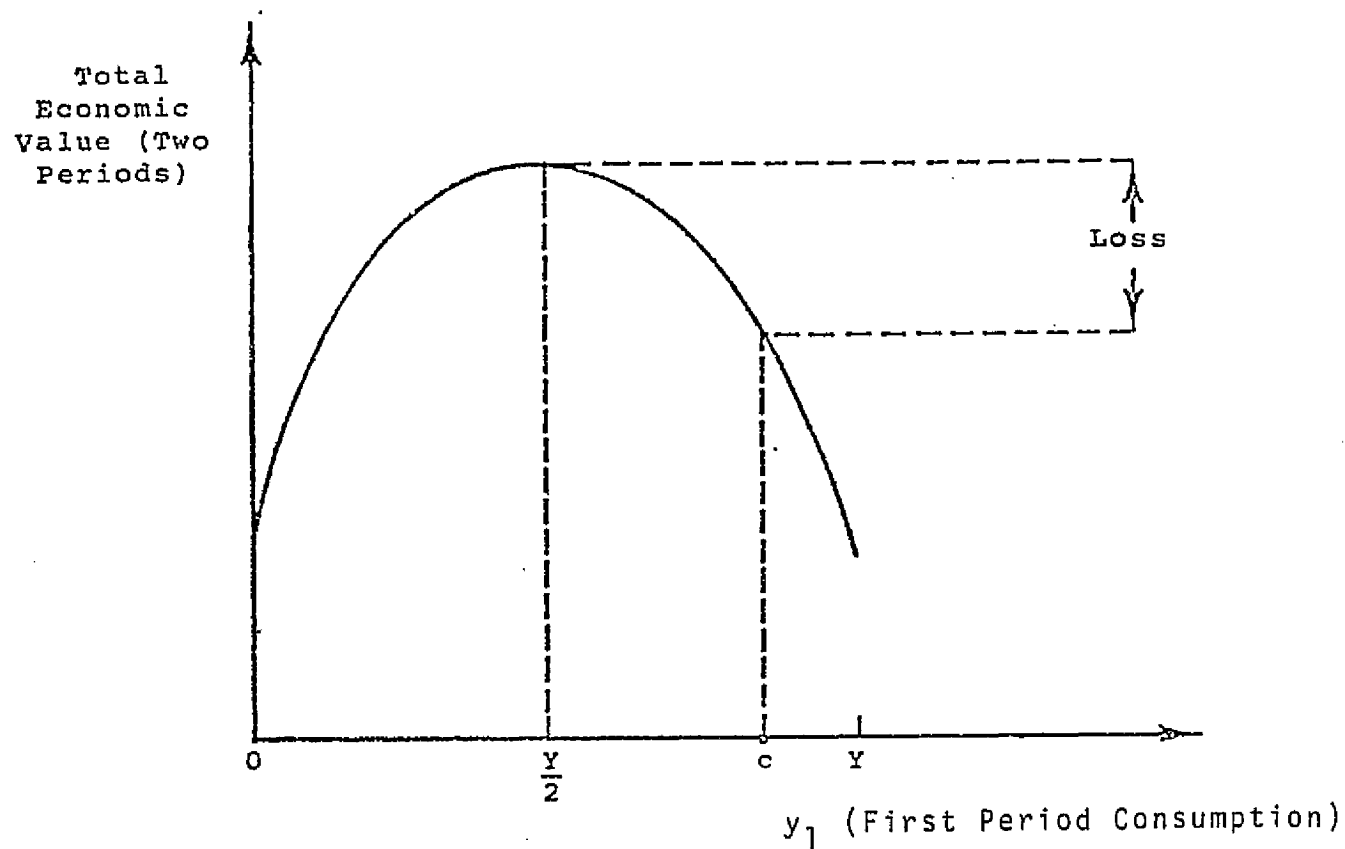


Figure 2.4 Economic Value as Function of Consumption Pattern — Two Periods

is represented by the decline of value from the maximum. For example, if the first period consumption is c (and the second is $Y-c$), the loss is as indicated by the horizontal dotted lines in Figure 2.4.

In a similar way, the economic loss associated with nonoptimal spatial distribution is determined from demand functions applying at the same time in different regions. Suppose that the demand functions in the United States and in the aggregated rest of the world are as shown in Figure 2.5. The horizontal axis is labeled increasing from left to right to indicate United States consumption, and increasing from right to left to indicate consumption in the rest of the world. The total world consumption, Y is represented by the length of the entire horizontal axis. The price axis for the United States demand function is on the left of the graph and the price axis for the rest of the world demand function is on the right of the graph. The nonoptimal situation portrayed in Figure 2.5 involves consumption y in the United States with a resulting gross economic value in the United States as indicated by the shaded region on the left. The consumption in the rest of the world is $Y-y$, with a gross economic value as indicated by the shaded area on the right. As is easily seen from the

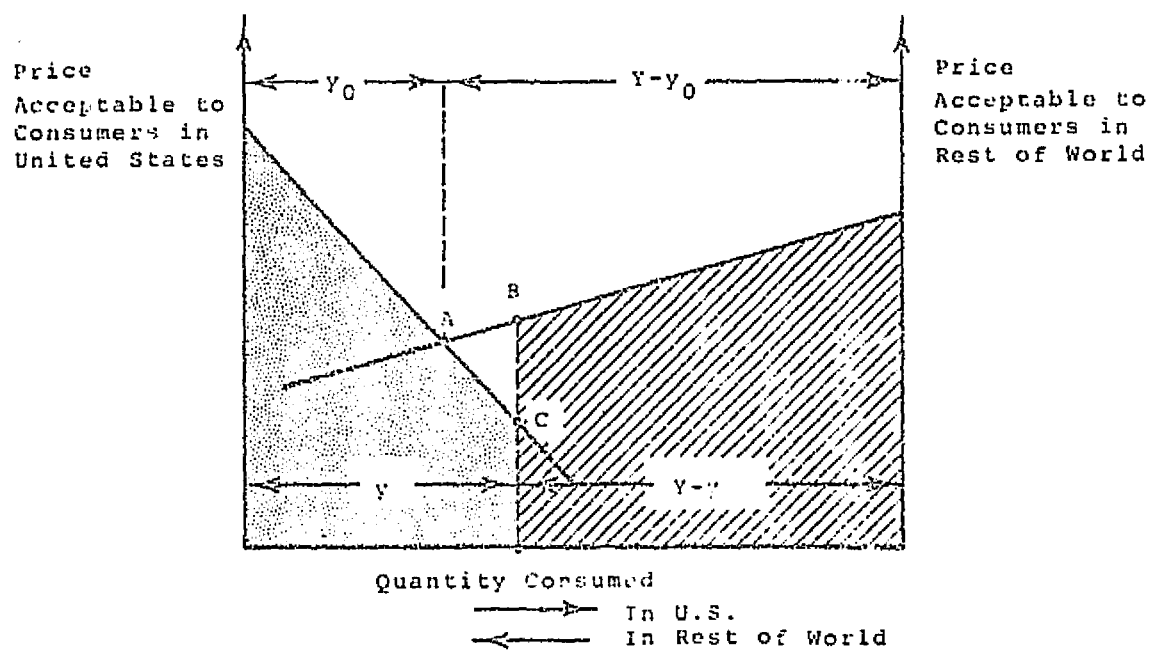


Figure 2.5 Demand Functions in Two Regions--
Nonoptimal Consumption Pattern

graph, the optimal division of consumption from the point of view of consumers in the entire world is y_0 in the United States and $Y - y_0$ in the rest of the world, and the loss of economic value associated with the pattern $(Y, Y - y)$ is the area of the triangle ABC. Just as in the temporal distribution example, the total economic value as a function of the consumption in one region can be graphed as a parabola. This is shown in Figure 2.6.

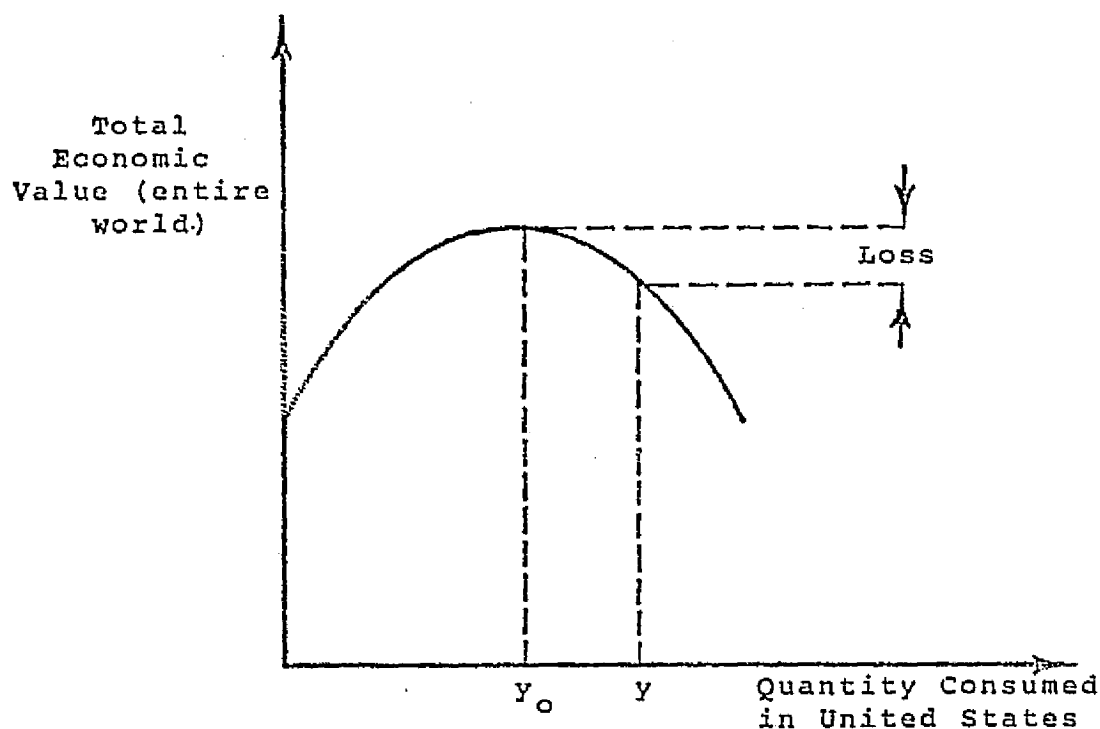


Figure 2.6 Economic Value as Function of Consumption Pattern — Two Regions

If information is adequate on supply patterns, if the market for wheat is unregulated, and if transportation and storage are inexpensive enough, the nonoptimal spatial and temporal distribution patterns discussed above will not occur. This is because the nonoptimal distribution patterns provide arbitrage opportunities, which cannot survive in a free market.

In other words, under the conditions listed above, the distribution decisions made by free market agents are such as to maximize the economic value associated with the consumption pattern. When production information is imperfect, some consumption or trade decisions must be made before it is possible to know their consequences for the total consumption pattern. When storage or transportation costs are significant, one must replace considerations of gross economic value with economic value net of such costs. In the presence of these complications, the free market still makes optimal decisions, but now the meaning of optimal is a bit more subtle. An optimal decision now is one which maximizes the mean of the probability distribution of discounted net present value of consumption, conditional on the information currently available. Thus, when information is imperfect, "optimal" means the best attainable on the average under the given

conditions of information. And numerically, we would expect that the mean present value attained under optimal decision making improves with information.

The model developed in Chapter 3 provides a means to calculate this improvement in value with information.

3. THE DISTRIBUTION MODEL

3.1 Control Process Formulation

The basic approach used in this study to determine the economic value of information on crop production is to model the distribution process as a dynamic control process. An overview of this basic model is given in Figure 3.1. Production of wheat in the United States and in the rest of the world is described by a stochastic production process which is considered fixed and exogenous to the control process. This is indicated by the oval block to the upper left. The three interrelated blocks constitute the distribution system which converts the worldwide production pattern into the worldwide consumption pattern. Also fixed and exogenous is the market demand model, which converts a given consumption pattern into economic value. This is shown on the middle right of Figure 3.1.

The distribution equation, shown as a rectangular block, is simply the linear relation describing how exports, production, and inventory adjustments affect the supply pattern (unconsumed supply in various regions). The system is subject to partial control through export decisions and inventory adjustments. The control is only partial since production has a random component. However, the application

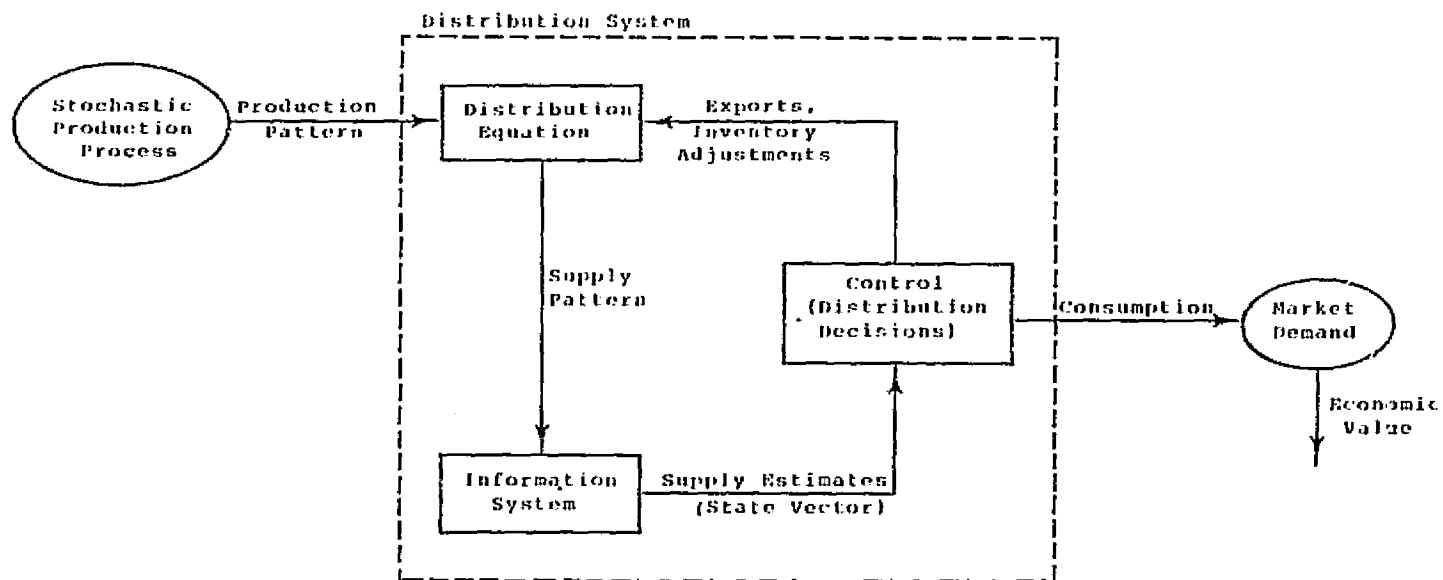


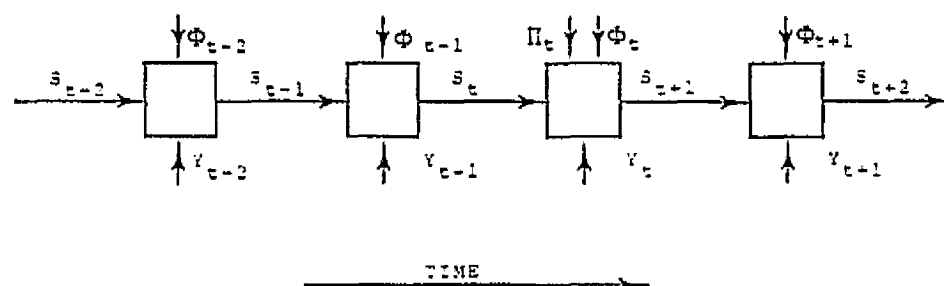
Figure 3.1 Overview of Feedback Control Model

of the control is made in the light of estimates of supply provided by an information system. Simultaneously with the decisions on exports and inventory adjustments, the control block produces the consumption pattern as output of the entire distribution system.

The use of the model outlined above in determining the value of production information is straightforward. One simply observes how the model output (economic value) changes in response to selected changes in the information system.

The dynamic dimension of the basic model structure is shown in Figure 3.2, together with notation for some of the important variables. The mathematical development of the model is based on the analysis of several functions of the state vector, which is the output of the information system in Figure 3.1. This state vector changes through time as indicated in Figure 3.2. A crop year is divided into decision periods 1, 2, ..., t , $t+1$, ..., m .

The state vector S_{t+1} at time $t+1$ depends on the state vector S_t at time t , on the decision or control vector Y_t at time t , and on a stochastic term ϕ_t . We assume that the decisions made are optimal in the sense that the mean discounted economic value of the consumption pattern indefinitely into the future is maximized, based on the information available.



s_t : State Vector at time t	—	Vector of supply estimates in U.S. and in the rest of the world
γ_t : Control Vector at time t	—	Vector of consumption in U.S., in rest of world, and exports.
Φ_t : Stochastic Term	—	Vector of change in production estimates in U.S. and rest of world.
Π_t : New Production	—	Vector of new production (only present at annual intervals).

Figure 3.2 Dynamic Dimension of Model —
Development of State Vector

The mathematical details of this optimization procedure are given in Sections 3.1 to 3.6, and Sections 3.7 to 3.9 describe the use of the model for benefit estimates.

3.2 Dynamic Structure of System

State Vector

The state of the system at time t is specified by the state vector

$$x_t = \begin{pmatrix} x_{t1} \\ x_{t2} \end{pmatrix} .$$

The first component, x_{t1} , is the mean at time t of the exporter's inventory. Notice that x_{t1} is not the inventory itself at time t , but the mean value of that quantity according to information available at time t . Similarly, the second component, x_{t2} , is the mean at time t of the importer's inventory.

Control Vector

The system is influenced by a control vector (decision vector) of three components

$$Y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} .$$

The first component, y_1 , is the current period's consumption in the exporting unit. The second component, y_2 , is the current period's consumption in the importing unit. The third component, y_3 , is the quantity exported in the current period. It is assumed that exports have a transit time of one period, so that the quantity y_3 becomes available for consumption or storage in the importing unit at the beginning of the next period.

State Transformation

Besides being influenced by the control vector, Y , the system is influenced by production and by new information on production. Information comes in the form of changes in production forecasts or estimates. A production forecast or estimate at time t is assumed to be the mean value of production based on all information available at time t . Let ϕ_{t1} be the change from time t to time $t+1$ of the production forecast (or estimate) for the current crop year in the exporting unit. If time t is the beginning of the last period in the crop year, then ϕ_{t1} will include the production forecast (of time 1) for the next crop year. Similarly, let ϕ_{t2} be the change from time t to time $t+1$ of the production forecast (or estimate) for the current crop year in the importing unit. We use the vector notation

$$\phi_t = \begin{pmatrix} \phi_{t1} \\ \phi_{t2} \end{pmatrix} .$$

Now we can write the state transformation, giving the state vector at time $t+1$ as a function of the state vector at time t and the control (decision) vector:

$$x_{(t+1)1} = x_{t1} + \phi_{t1} - y_1 - y_3 ,$$

$$x_{(t+1)2} = x_{t2} + \phi_{t2} - y_2 + y_3 .$$

In matrix notation, this can be written

$$x_{t+1} = x_t + MY + \phi_t , \quad (1)$$

where

$$M = \begin{pmatrix} -1 & 0 & -1 \\ 0 & -1 & 1 \end{pmatrix} .$$

3.3 The Value Function

A value function, W_t , is defined for each time t . W_t is a real value function on the state space, that is, the two dimensional space of possible values of the state vector x_t . $W_t(x_t)$ is the maximum, over the possible choices of y at times $t, t+1, t+2, \dots$, of the discounted mean economic value

of consumption minus transportation costs in periods $t, t+1, t+2, \dots$ for the pair of units. The present value is taken to an infinite horizon. The mean is with respect to the uncertainty in the coming changes $\phi_t, \phi_{t+1}, \phi_{t+2}, \dots$, in production estimates in the future. By economic value of consumption in a period is meant the integral under the demand curve for that period from zero to the amount consumed. This global value function W_t will form the basis for our analysis of optimal decision making and the value of improved information.

The state transformation at time t changes the state vector X_t according to Equation (1). In addition, it results in an increment $w(Y)$ to the value function W_t . This increment is

$$w(Y) = \alpha y_1^2 + \beta y_1 + \gamma y_2^2 + \delta y_2 - \tau y_3 - \tau_1 y_3^2.$$

We will also be concerned with the incremental economic value

$$u(Y) = \alpha y_1^2 + \beta y_1 + (2\alpha y_1 + \beta) y_3,$$

to the exporting unit in the period beginning at time t .

These equations are based on linear demand functions

$$\text{price} = 2\alpha(\text{quantity}) + \beta$$

in the exporting unit and

$$\text{price} = 2\gamma(\text{quantity}) + \delta$$

in the importing unit. Exports are purchased at the price

$$2\alpha y_1 + \beta$$

prevailing at the time of order in the exporting unit.

Transportation costs are paid at the same time by the importing unit; the costs increase with volume according to the formula

$$\text{Cost} = \tau y_3 + \tau_1 y_3^2.$$

Storage costs other than interest are omitted because they are small in the case of our current applications (Interest costs are accounted for by the factor e in Section 3.4, Equation 4).

It will be convenient to use matrix notation in the expressions for w and u . Accordingly, let

$$A = \begin{pmatrix} \alpha & 0 & \alpha \\ 0 & 0 & 0 \\ \alpha & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} \beta \\ 0 \\ \beta \end{pmatrix},$$

$$E = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \gamma & 0 \\ 0 & 0 & -\tau_1 \end{pmatrix}, \quad F = \begin{pmatrix} \beta \\ \delta \\ -\tau \end{pmatrix}.$$

Now we can write

$$w(Y) = Y^*EY + Y^*F , \quad (2)$$

$$u(Y) = Y^*AY + Y^*B , \quad (3)$$

where the asterisk indicates matrix transposition.

The equations (1) and (2) specify the structure of the control system under analysis. The development through time of the state vector and the value functions is determined by a specification of the stochastic terms ϕ_{tj} and the controls Y . If we assume that Y is determined at each time by an optimization rule, then the value functions depend only on the stochastic terms and can be determined from the solution to a functional equation called the principle of optimality.

3.4 The Functional Equation

The principle of optimality for the control system described above is

$$W_t(X_t) = \max_Y \left[w(Y) + \rho \overline{W_{t+1}(X_{t+1})} \right] . \quad (4)$$

Here, ρ is the discount factor for one period, and the bar indicates the mean value with respect to the random variables ϕ_{t_1}, ϕ_{t_2} .

The solution to this functional equation will provide both the value functions W_t and the decision rules $\hat{Y}_t$ for each time t . The decision rule is a function giving the optimal control vector $\hat{Y}_t(X)$ for each value X of the state vector. Knowing these, we will be able also to calculate the value functions $u(Y)$ in the exporting unit, and the value of improved production information to the importing unit, to the exporting unit and to the pair of units.

Two basic approaches to functional equations of this type have been developed in the literature of Dynamic Programming. The first is to represent the functions by their values at selected points of the state space and to do the maximizations and mean value calculations by straightforward numerical methods. The second approach is to represent the value functions by the parameters of chosen functional forms and to use the knowledge of these forms to facilitate the maximizations and mean value calculations.

We apply here the second approach, parametric representation, using second degree polynomials to represent the value functions. This will allow us to do the maximizations and mean value calculations very simply. Accordingly, we assume that the value functions have the following form:

$$W_t(X) = X*Q_tX + X*P_t + r_t .$$

Here, Q_t is a symmetric 2×2 matrix, P_t is a column vector of 2 components, and r_t is a scalar.

3.5 Backward Induction

The objective now is to calculate the coefficients Q_t , P_t , r_t , given values for Q_{t+1} , P_{t+1} and r_{t+1} .

We begin by expanding $\overline{W}_{t+1}(X_{t+1})$, which appears in the right side of Equation (4).

$$\begin{aligned}
 \overline{W}_{t+1}(X_{t+1}) &= \overline{W}_{t+1}(X_t + MY + \phi_t) \\
 &= \overline{(X_t + MY + \phi_t) * Q_{t+1} (X_t + MY + \phi_t)} \\
 &\quad + (X_t + MY + \overline{\phi}_t) * P_{t+1} + r_{t+1} \\
 &= Y * M * Q_{t+1} MY + 2Y * M * Q_{t+1} (X_t + \overline{\phi}_t) \\
 &\quad + \overline{(X_t + \phi_t) * Q_{t+1} (X_t + \phi_t)} + Y * M * P_{t+1} \\
 &\quad + (X_t + \overline{\phi}_t) * P_{t+1} + r_{t+1} .
 \end{aligned}$$

Now using this expression and Equation (2), Equation (4) becomes

$$\begin{aligned}
W_t(X_t) = \max_Y \bigg\{ & Y^* \left[E + \rho M^* Q_{t+1} M \right] Y \\
& + Y^* \left[F + \rho M^* (P_{t+1} + 2Q_{t+1} (X_t + \bar{\Phi}_t)) \right] \\
& + \rho \left[X_t^* Q_{t+1} X_t + 2X_t^* Q_{t+1} \bar{\Phi}_t + \overline{\Phi_t^* Q_{t+1} \Phi_t} \right. \\
& \left. + (X_t + \bar{\Phi}_t) P_{t+1} + r_{t+1} \right] \bigg\}. \tag{5}
\end{aligned}$$

The maximization here is subject to constraints. Each component of Y must be nonnegative, and the sum of exports and consumption in the exporting unit must not exceed the inventory in the exporting unit. Finally, the consumption in the importing unit must not exceed the inventory there. Formally, the constraints are,

$$y_1 \geq 0, y_3 \geq 0,$$

$$x_2 \geq y_2 \geq 0,$$

$$x_1 \geq y_1 + y_3.$$

We use a standard quadratic programming algorithm to find the constrained maximum of the quadratic form in (5). Because of the influence of the constraints, the

resulting value function W_t is not necessarily of the second degree polynomial form assumed for W_{t+1} . However, it can be approximated by such a form. Our procedure is to evaluate W_t on a grid of 25 points in the state space and then determine the best least squares fit of a second degree polynomial to the function W_t .

The coefficients of this best fit are then used to form the matrices Q_t , P_t and r_t , completing the backward induction step of the solution of Equation (4).

3.6 Iterative Solution

Equation (4) represents a chain of relations between functions W_t for successive times t . Thus, with the aid of the calculations discussed in Section 3.5, we can find W_t at all earlier times, provided that we have W_T corresponding to some horizon time T . But if the horizon is far enough in the future, that is if $T \gg t$, W_t will be nearly independent of W_T . Further, the value functions will be periodic, since the demand functions and the state transformations are periodic, with period one year. Thus, a practical way of solving (4) is to begin with an arbitrary starting function W_T , and calculate backwards until the corresponding value functions of successive years are as close to equal as desired. At this point, we will have m value functions and decision rules, where m is the number of periods per year.

3.7 Simulation for Probability Distributions of Value, Exports and Inventories

The decision rules $\hat{Y}_t(X_t)$, $t = 1, \dots, m$, specify the consumption and export decisions appropriate to any possible state at time t . Thus, they provide a method for simulation of the production information and trade system through as many years as desired. Starting at any convenient value of the vector X_1 , one can apply the state transformation

$$X_2 = X_1 + M\hat{Y}_1(X_1) + \Phi_1,$$

obtaining Φ_1 by random sampling from its distribution. The consequent economic value increments $w(\hat{Y}_1(X_1))$ and $u(\hat{Y}_1(X_1))$ are determined from Equations (2) and (3). The process is then repeated. Continuing indefinitely, one can determine the means, variances, or other properties of the distribution of annual value in the exporting unit, and in the pair of units, and the same descriptors of the exports in each period of the year, and the stocks in the two units.

To determine the value of one information system, compared to another, one determines for each system the probability distributions of $\Phi_1, \dots, \Phi_m$. Then one determines the mean economic value over many years in the exporting unit and in the pair of units, using the procedures of Sections 3.4.4,

3.4.5 and 3.4.6. The value of information is the difference of the mean economic values for the two cases.

3.8 Iteration to Determine Grid for Approximation of Value Function.

The decision rule calculated by solution of the functional equation (4) can be "fine tuned" by an iterative procedure involving the location of the 25 equiprobable grid points used to approximate the value functions W_t . It is desirable that this quadratic approximation be particularly accurate in the region of the state space most often encountered. But in advance of solving the equation, the critical region is not known. However, one can make an initial choice of the grid for each period of the year, then solve Equation 4, then use the resulting decision rule in a simulation of the system, tabulating the frequency distribution of the state vector occurring in each period. Then one can make a new choice of the grid, corresponding to this frequency distribution, and can repeat the entire procedure. This procedure has been found to be convergent, and leads to an improved decision rule, as indicated by higher mean values of W_t .

Specifically, we have used a uniform rectangular grid as shown in Figure 3.3. The central point (ξ_{t1}, ξ_{t2}) is called the presumptive state for period t . It is initially chosen on the assumption that consumption and

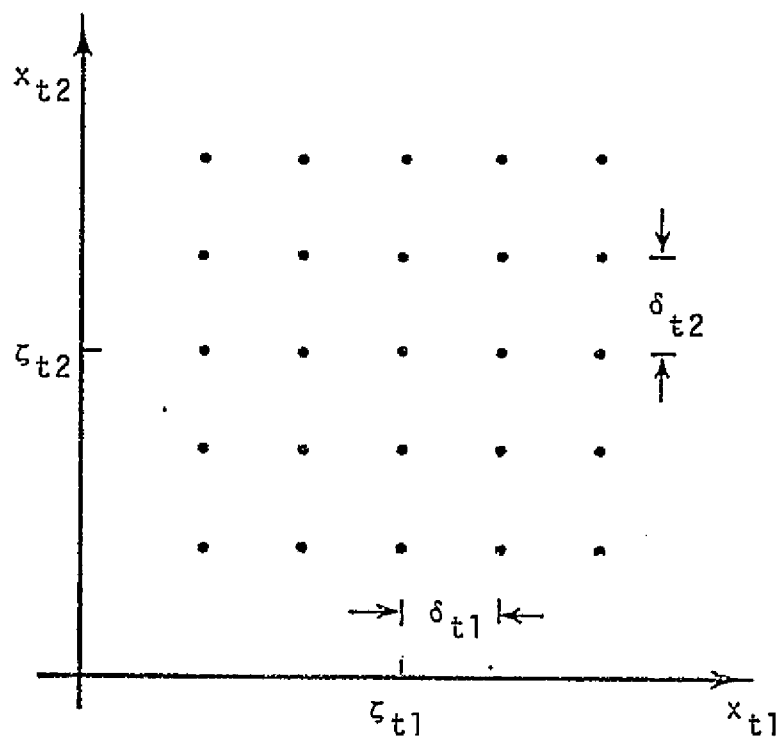


Figure 3.3 Grid for Approximation of Value Function

exports will be uniform throughout the year and will be consistent with the demand functions in annual form.

The annual demand functions in the exporting unit and the importing unit are

$$\text{price} = \frac{2\alpha}{m} (\text{quantity}) + \beta ,$$

$$\text{price} = \frac{2\gamma}{m} (\text{quantity}) + \delta ,$$

respectively. Applying these to the total annual production Π with annual exports e , the equilibrium condition is

$$\frac{2\alpha}{m} (\pi_1 - e) + \beta = \delta + \frac{2\gamma}{m} (\pi_2 + e) - \tau - \tau_1 e .$$

Solving for e , the presumptive annual exports are

$$e = \frac{2\alpha\pi_1 - 2\gamma\pi_2 + (\tau + \beta - \delta)m}{2\alpha + 2\gamma - \tau_1}$$

For the initial formation of the grid, we estimate that the buffer inventories, B , are equal to the standard deviation of the remaining supply uncertainty one period before the new harvest. Thus, in the exporting unit they are

$$b_1 = \sqrt{\phi_{i1}^2} ,$$

where i is the index of the period before the exporter's harvest. In the importing unit the buffer inventories are estimated as

$$b_2 = \sqrt{\phi_{j2}^2} ,$$

where j is the index of the period before the importer's harvest.

The presumptive state variables ξ_{t1} and ξ_{t2} are thus assumed to decline linearly from their maximum values $b_1 + \pi_1 - e$ and $b_2 + \pi_2 + e$ to b_1 and b_2 . The maxima for the two units may of course occur in different periods.

In the initial grid, the intervals δ_{t1} and δ_{t2} are chosen to give a standard deviation of one half the mean in each component. Thus, for $i = 1, 2$, we have

$$\sqrt{\frac{2\delta_{ti}^2 + 2(2\delta_{ti})^2}{5}} = \sqrt{2}\delta_{ti} = \frac{1}{2} \xi_{ti} ,$$

or

$$\delta_{ti} = 0.354 \xi_{ti} .$$

In the next iteration, after the simulation of the system, a new grid is selected. For this grid, the central point (ξ_{t1}, ξ_{t2}) at time t is just the mean value from the simulation of the state vector at time t , and the intervals δ_{t1} and δ_{t2} are chosen such that

$$\delta_{ti} = \frac{1}{\sqrt{2}} \sigma_{ti}$$

where $(\sigma_{t1}, \sigma_{t2})$ is the standard deviation from the simulation of the state vector at time t .

4. RESULTS — DISTRIBUTION OF WHEAT

The model developed in Chapter 3 was applied to the pair of units — United States and aggregated rest of the world. A detailed presentation of the algorithm used is given in Appendix A. Decision rules appropriate for "current" information levels were calculated and applied in ten 30-year simulations. Significant quantities such as discounted present value to the U.S. exports, prices, consumption rates, and stock levels were recorded for each simulation, and then averaged over the ten simulations. Similarly, the decision rules appropriate for a "satellite" information system were calculated and applied in ten 30-year simulations. The same summary statistics were collected. Of most importance are the means over the ten simulations of the discounted present value to the U.S. Comparing these figures for the "current" and "satellite" systems, we determine the benefits of the improved information to the U.S.

The numerical descriptions given of "current" and "satellite" systems are approximate.* Though the "current" system corresponds roughly to the level of information in today's markets, and the "satellite" system reflects a specific LACIE target, neither accounts completely for the dynamics of information development throughout the growing season and the marketing year.

*The difficulties involved in an accurate quantitative analysis of existing or projected information systems are discussed in Volume 2 of this study.

4.1 Input Data

Table 4.1 gives the demand function parameters, the discount rate, the transportation time and cost parameters, and the average production figures used as input to our calculations.

Since our calculations are done in constant (1975) dollars, the discount rate used is lower than typical market interest rates, which are quoted in current dollars. The transportation cost parameters are based on estimates by Bradford and Kelejian [2].

The calculations discussed here are based on a division of the year into six periods. The beginning of the year is August 1 since this puts the bulk of the world's wheat harvest in the first period. The United States production averaging 50 million metric tons is treated as occurring in the sixth period, while the rest of the world production averaging 300 million metric tons is treated as occurring in the first period. Each information system considered provides the first information on the new crop at the beginning of the harvest period. Thus the first information on United States production comes in June and the first information on the rest of the world production comes in August. Regardless of the quality of earlier information, it is assumed that the true production for the crop year is known at the start of the final period

Table 4.1 Data Common to All Calculations		
Annual Discount Rate (%)	6.2	
Transportation Time (months)	2.0	
Transportation Cost (1975 \$)		
Cost = $\tau e + \tau_1 e^2$		
(e is annual exports in metric tons)		
τ	6.1	
τ_1	0.055	
	United States	Rest of World
Average Annual Production (millions of metric tons)	50.0	300.0
U.S. Price of Wheat (1975 \$/metric ton)		
Price = $Aq + B$		
(q is annual consumption in metric tons)		
A	-16.5	-1.75
B	495	749.7

of the crop year — April 1 in the United States and June 1 in the rest of the world. Thus we assume that market decisions are made as if the true production is known at the end of the year, perhaps by direct discovery of scarcity, if not by the revisions of published estimates.

Table 4.2 gives input data describing information system performance.

In the case of the "current" information system, it is assumed that before any information is available specific to the coming crop, the standard deviation of the probability distribution of the August 1 forecast of rest of the world production is 31.6 million metric tons. Of course the mean of this probability distribution is the average annual production of 300 million metric tons. These figures occur in the bottom row of Table 4.2, third column. After the August 1 forecast, no more information is produced until June 1, when the error in the August 1 forecast is discovered. The standard deviation of this error is also 31.6 million metric tons.

Thus, the "current" system resolves half the uncertainty in the rest of world production with the first forecast on August 1 and the residual uncertainty is resolved at the end of the crop year. The total uncertainty (in advance of any information specific to the crop) is characterized by a variance of

$$31.6^2 + 31.6^2 = 2000 \text{ (million tons)}^2$$

Table 4.2 Performance of Alternative Information Systems — Supply Variability of 1960's				
Period Beginning	Mean and Standard Deviation of Production Information Expected Next Period (millions of metric tons)			
	Current Capability		LACIE Targets	
	United States	Rest of World	United States	Rest of World
Aug 1	0,0	0,0	0,0	0,0
Oct 1	0,0	0,0	0,0	0,0
Dec 1	0,0	0,0	0,0	0,0
Feb 1	0,3	0,0	0,3	0,0
Apr 1	50,5.2	0,31.6	50,5.2	0,18.2
June 1	0,0	300,31.6	0,0	300,40.8

This is a measure of the total supply variability of the system. The standard deviation of rest of world supply variability is thus

$$\sqrt{2000} = 44.7 \text{ million tons,}$$

or 15% of the annual production.

LACIE targets call for determination of the year's production to within 10% with 90% probability at harvest — thus a 90% probability of error less than 30 million metric tons. If the error distribution is normal, this means the standard deviation of the error in the August 1 forecast is 18.2 million metric tons, or its variance is 333 (million tons)². Thus more of the total supply variance shows up in the first forecast, so that the variance of the first forecast itself is

$$2000 - 333 = 1667 \text{ (million tons)}^2$$

and its standard deviation is 40.8 million metric tons. These figures appear in the last column of Table 4.2.

For the United States crop, the two information systems perform equally well. Here the total variability is represented by a variance of 36 (million tons)², or a standard deviation of 12% of the mean. The first forecast (June 1) has an error variance of 9 (million tons)², and

its own variance is 27 (million tons)². The standard deviation of first forecast error is 6%, and there is no new information until the true production is discovered in the period beginning April 1.

4.2 Benefits

4.2.1 Summary of Results

Table 4.4 presents a summary of the results of the model applied to the data of Section 4.1. The system identified as "LACIE Targets" provides an annual benefit of \$108 million to the United States. As shown in Table 4.4, the benefit is associated with a reduction in price fluctuations, both in the United States and in the rest of the world. The table also indicates a small change in mean prices, but this is insignificant, and may be only "noise" in the simulations. In addition to the reduced price fluctuations, the United States benefit is related to the patterns of export flow within the year, which will be discussed in the next section. Here we observe that mean annual exports are essentially the same in the two cases, but the standard deviation of exports is somewhat higher with the improved information. Table 4.4 also shows mean buffer inventories. It is interesting to notice that the primary effect of improved information on buffer inventories is that they tend to shift toward the United States, though the total quantity carried over

remains about the same. This indicates that the optimal buffer inventory level is determined only by the actual total variability of the system, which is not reducible through information (unless production effects are considered).

4.2.2 Details on Results

The summaries of the previous section represents the average results of 10 simulations each for the "current" and "LACIE" systems. The individual simulations cover a period of 30 years, starting with buffer stocks at zero. The graphs of Figures 4.1, 4.2, ..., 4.14 indicate some of the results of just one of the 10 simulations for each of the systems.

These graphs show the changes in key quantities in the United States and the rest of the world for the first 15 years of the 30-year simulation. The first four graphs, in Figures 4.1, 4.2, 4.3, and 4.4 show the actual stocks compared with the state variables, or the current estimate of stocks. Thus these portray the current stock uncertainty during the course of the simulation.

The next four graphs, in Figures 4.5, 4.6, 4.7, and 4.8, show prices together with actual stocks and changes in production estimates. These changes in production estimates are the random inputs which "drive" the simulations. As discussed in Section 4.1, new information on United States production comes in June and the following April

Table 4.4 Benefit Summary -- Distribution of Wheat										
	Annualized Mean Present Value @ 6.2% to United States, \$ millions, 1975	Spot Prices, 1975 \$/metric ton				Annual Exports, millions of metric tons		Mean Buffer Stocks, millions of metric tons		
		United States Mean	United States Std.Dev*	Rest of World Mean	Rest of World Std.Dev*	Mean	Standard Deviation	United States	Rest of World	Total
Current Capability	12463	160.4	50.8	172.6	58.2	30.0	8.9	10.5	67.4	77.9
LACIE Targets	12571	158.9	34.7	172.9	48.7	29.7	10.6	15.2	63.2	78.4
Difference (LACIE-Current)	108	-1.5	-16.1	0.3	-9.5	-0.3	1.7	4.7	-4.2	0.5
*The standard deviation is that of the entire time series of prices in the simulation, the time series interval being two months.										

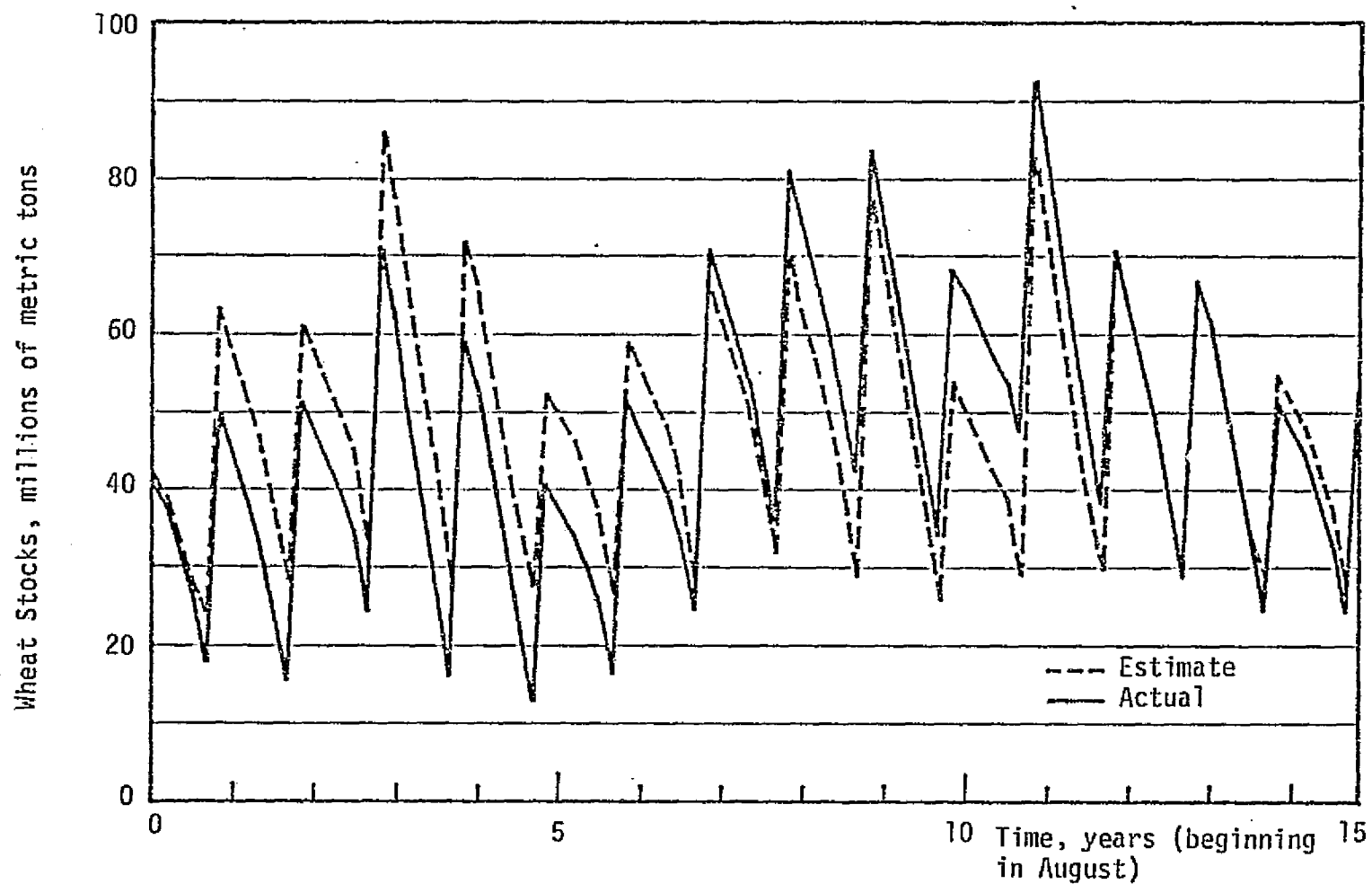


Figure 4.1 A Simulation Case--United States Stocks and Estimates of Stocks. Information System Meeting LACIE Targets.

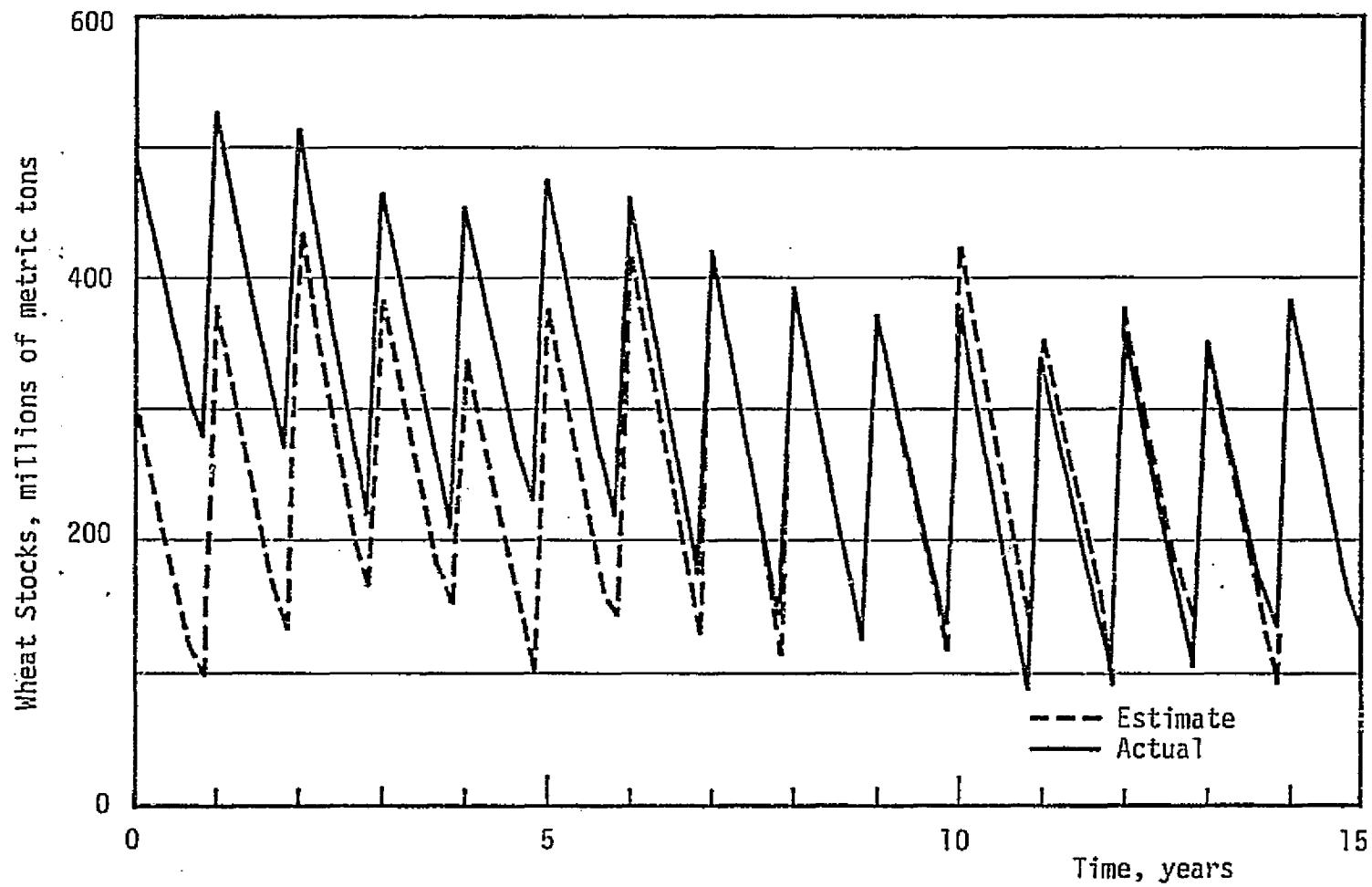


Figure 4.2 A Simulation Case--Rest of World Stocks and Estimates of Stocks. Information System Meeting LACIE Targets.

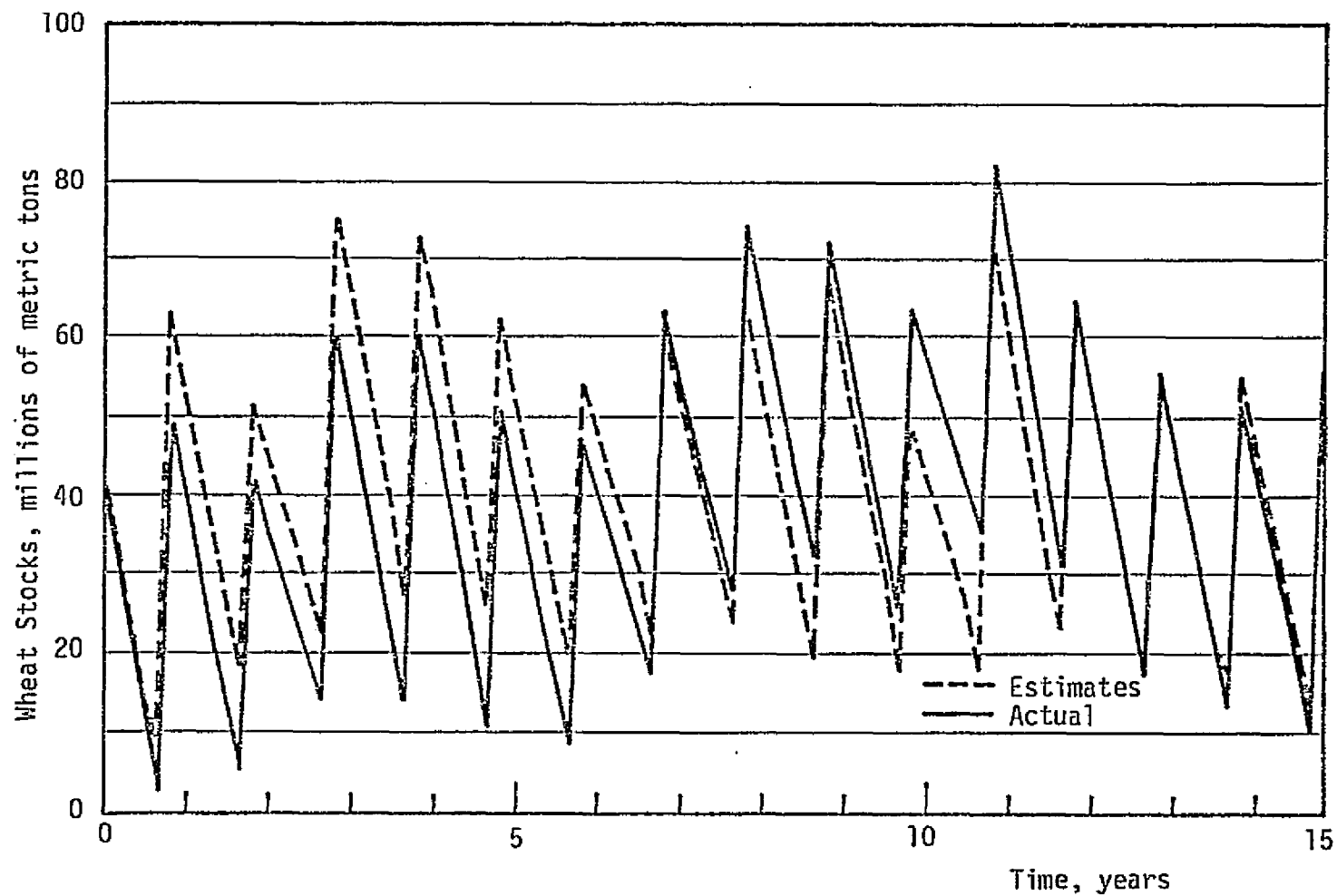


Figure 4.3 A Simulation Case--United States Stocks and Estimates of Stocks. Current Information System.

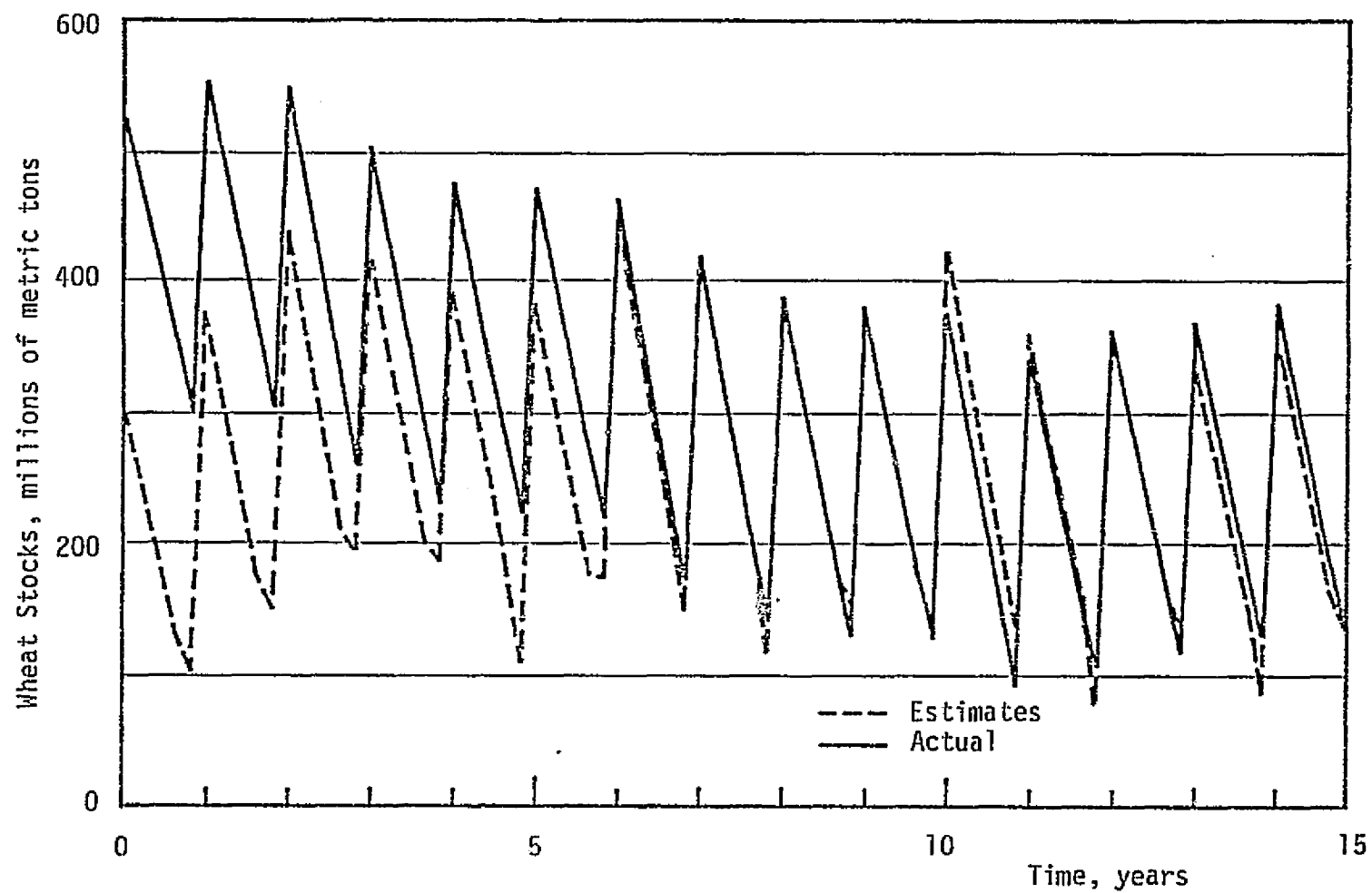


Figure 4.4 A Simulation Case--Rest of World Stocks and Estimates of Stocks. Current Information System.

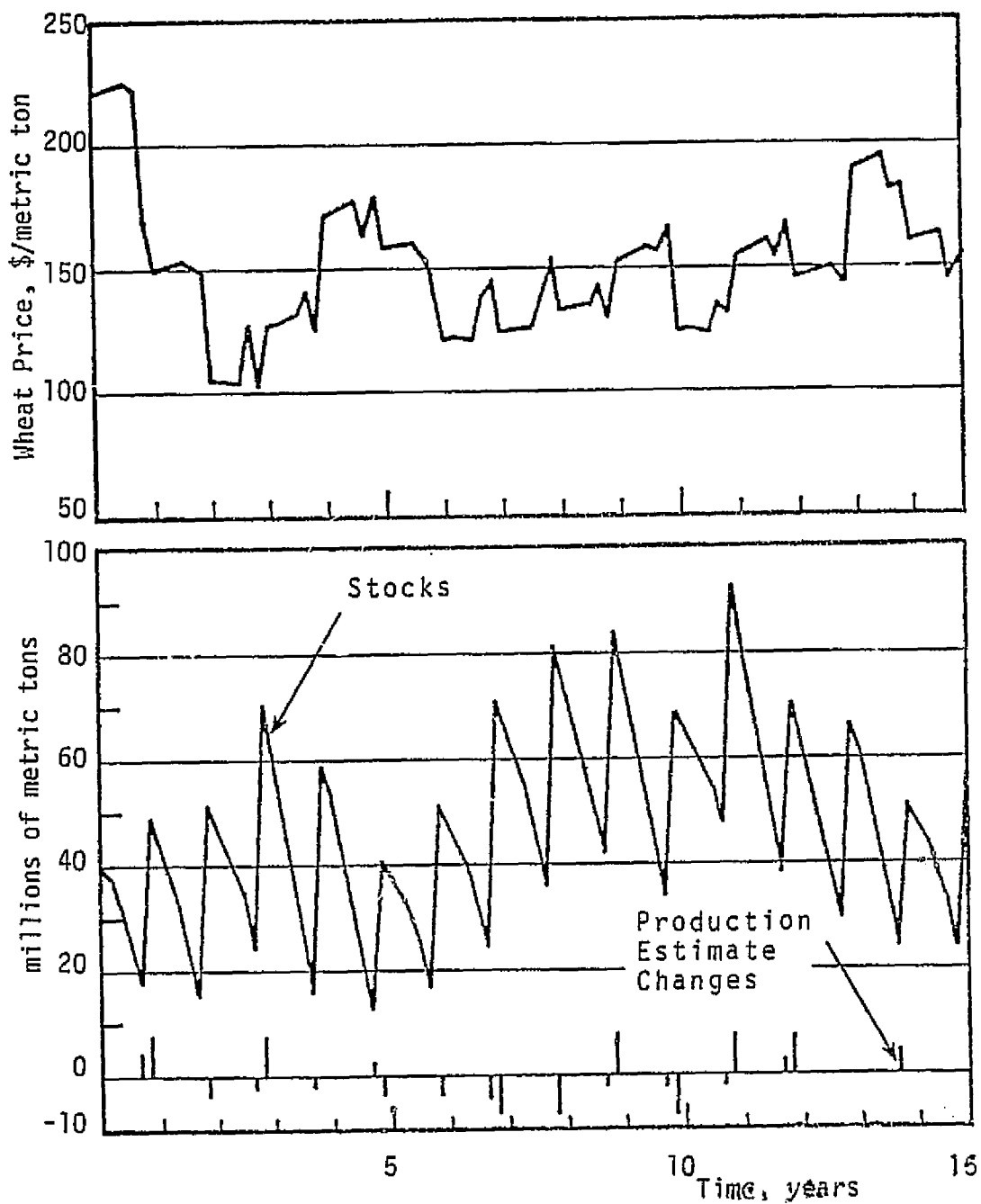


Figure 4.5 A Simulation Case--United States Prices, Stocks, and Production Estimate Changes. Information System Meeting LACIE Targets.

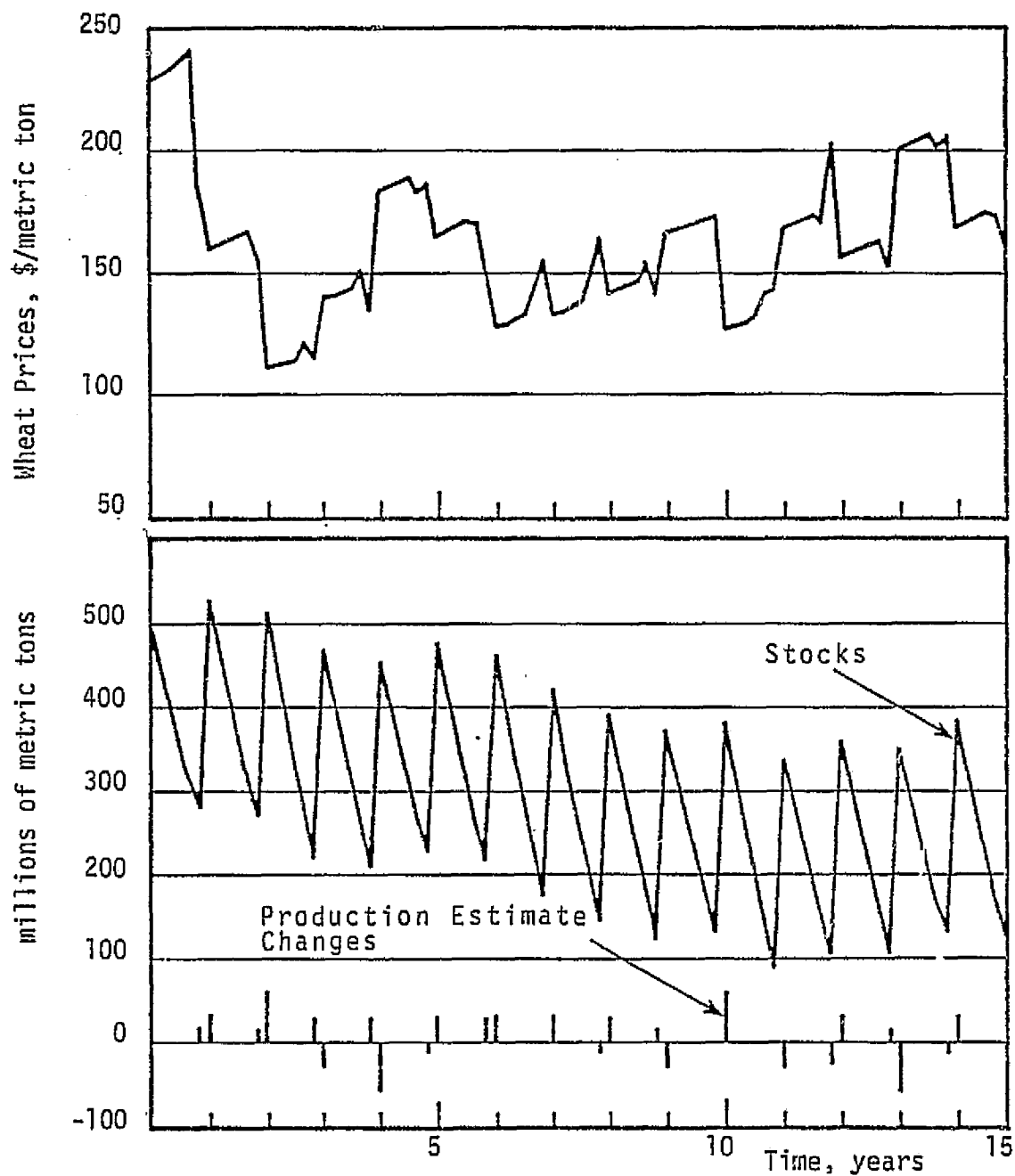


Figure 4.6 A Simulation Case--Rest of World Prices, Stocks, and Production Estimate Changes. Information System Meeting LACIE Targets.

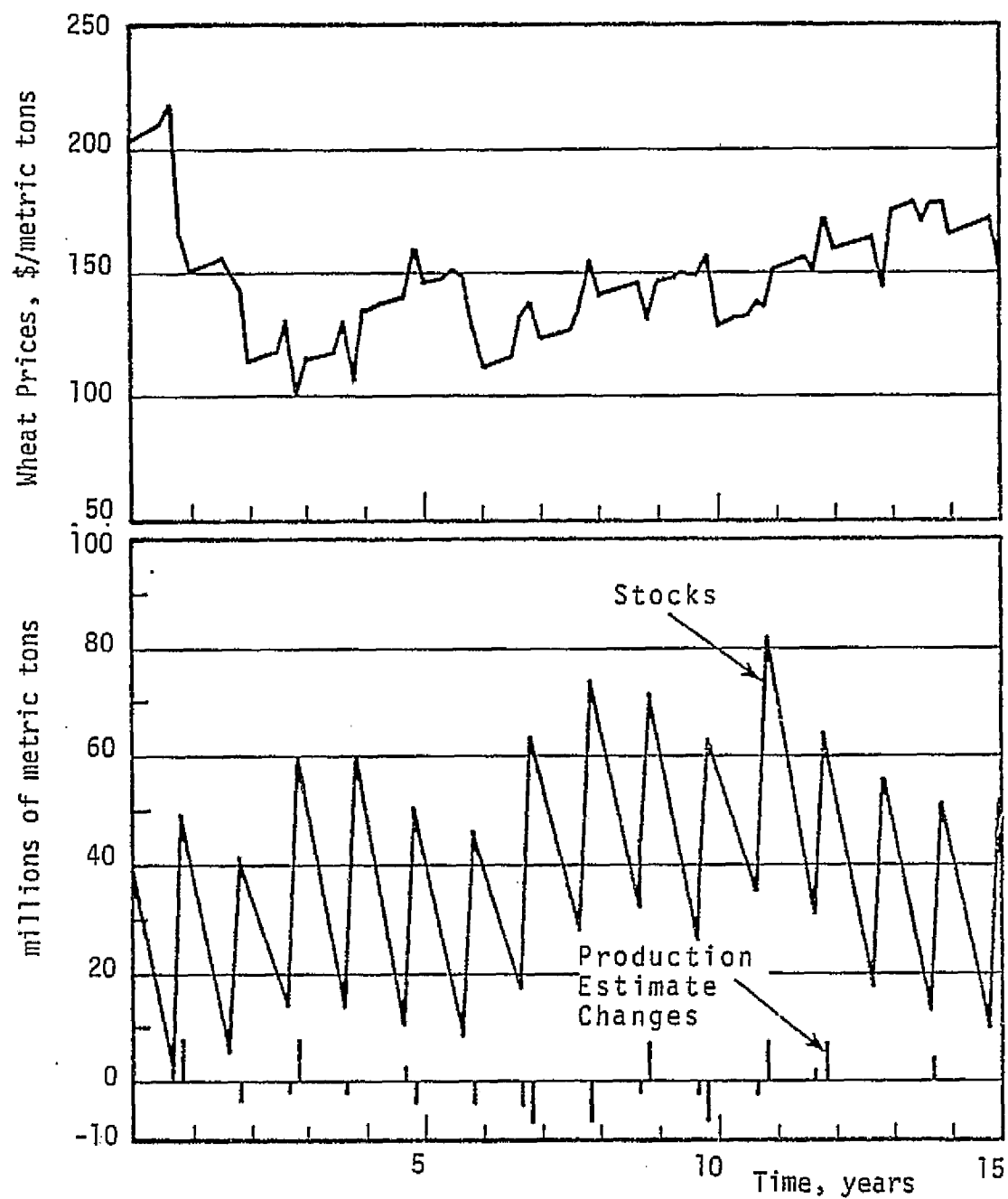


Figure 4.7 A Simulation Case--United States Prices, Stocks, and Production Estimates Changes. Current Information.

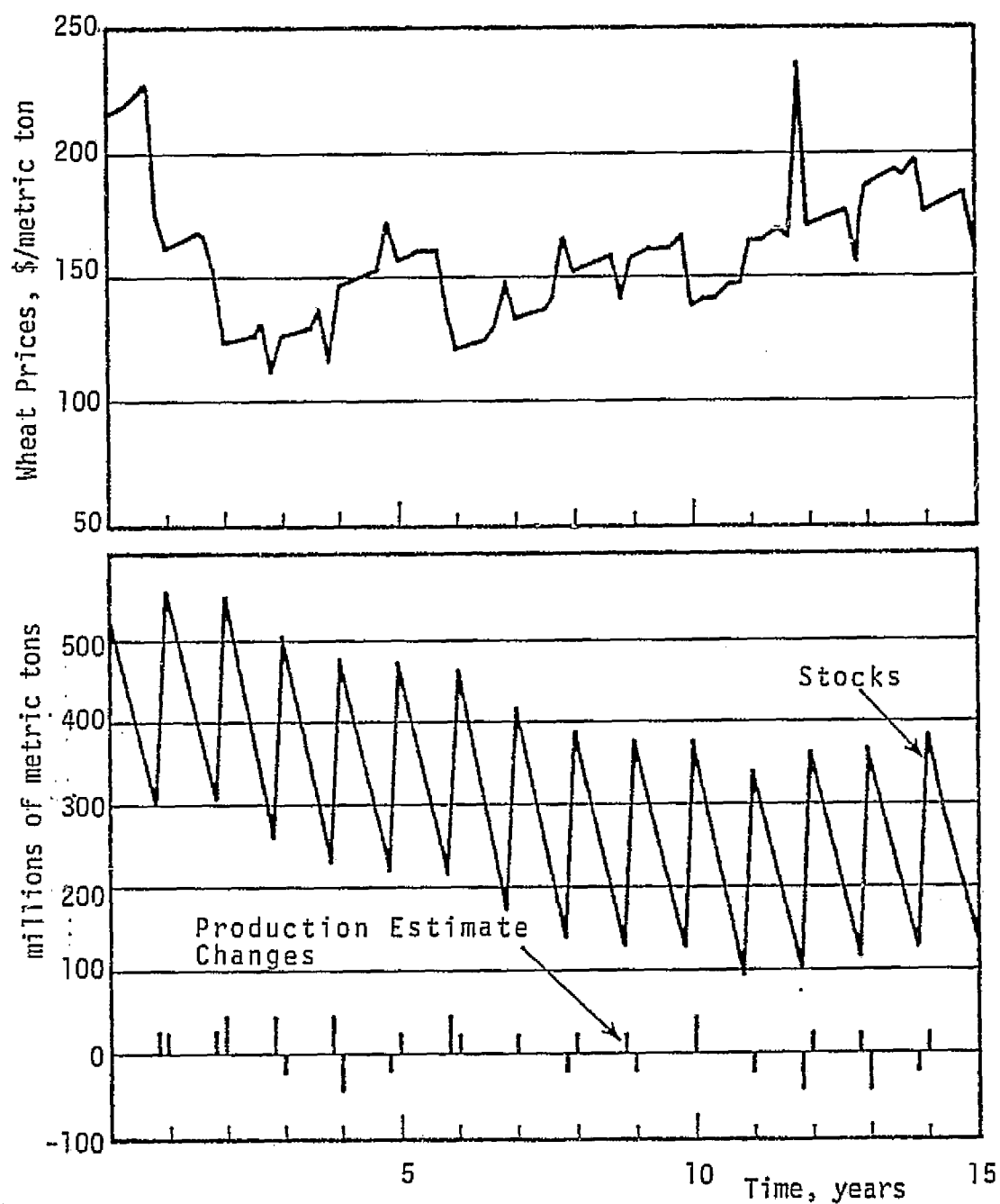


Figure 4.8 A Simulation Case--Rest of World Prices, Stocks, and Production Estimate Changes. Current Information.

while new information on rest of the world production comes in August and the following June. These graphs clearly show the response of prices and stocks to new information.

In Figures 4.9, 4.10, 4.11, and 4.12 are shown the consumption patterns together with the state variables (stock estimates). These are shown together because the consumption rates are regarded in our model as decision variables responding directly to the state variables. It is evident from these graphs that the fluctuations in consumption rates are rather small. This is to be expected, since the elasticities of demand for wheat in both the U.S. and the rest of the world are quite low.

The final graphs, in Figures 4.13 and 4.14, show the consumption rates and exports for the two information systems. It is noteworthy that the export pattern is quite different in the "LACIE" case from the "current" case. There are more periods of little or no exports as well as occasional periods of very large exports. The large export periods tend to occur when prices are rather high. Qualitatively, what is occurring is a more effective use of international trade as a consumption smoothing device, made possible by better advance information of the need. Prices in the United States are highest on the average in the fifth period (April 1 - June 1), just before the new United States harvest. With the "current" information

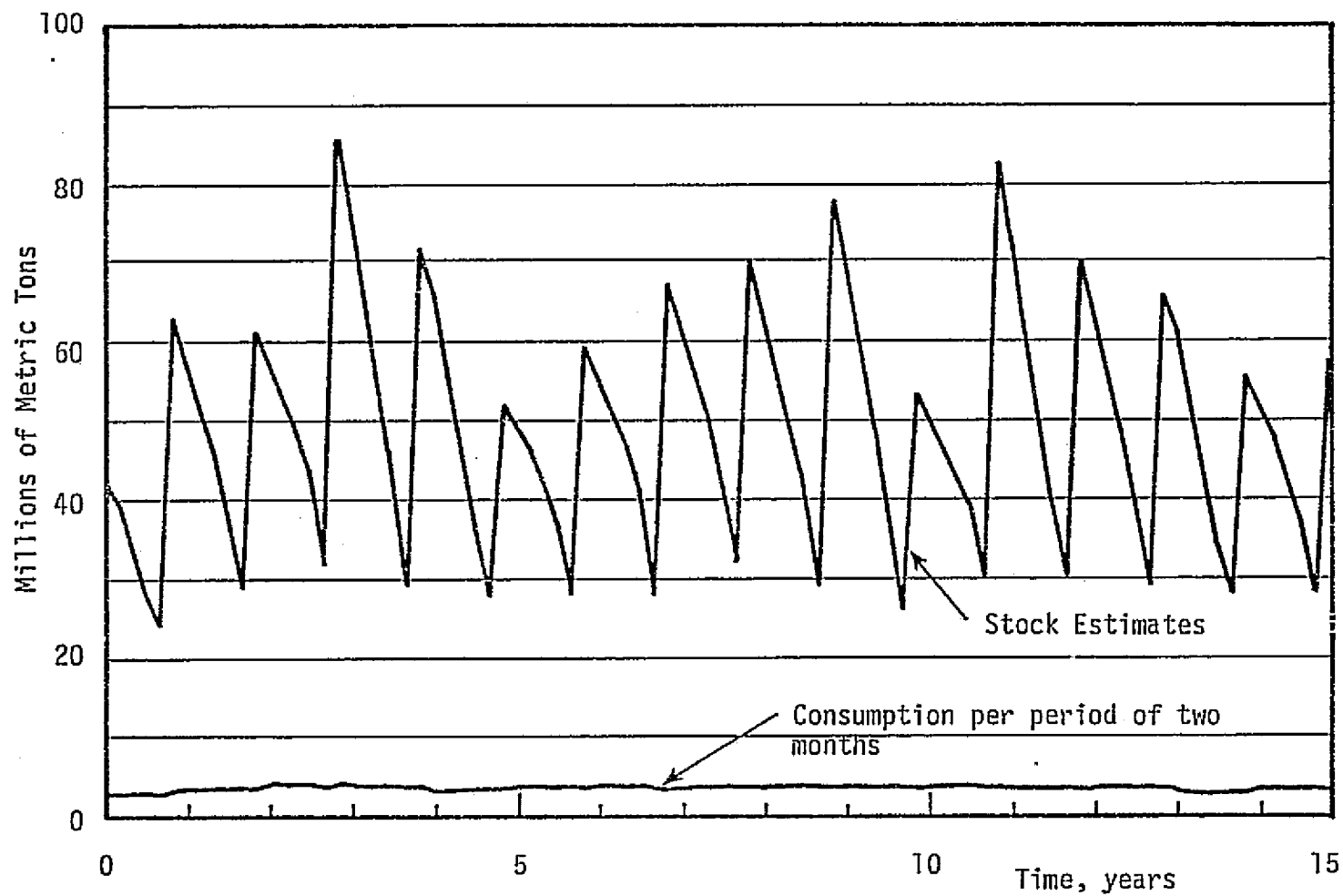


Figure 4.9 A Simulation Case--United States Estimates of Stocks, Consumption. Information System Meeting LACIE Targets.

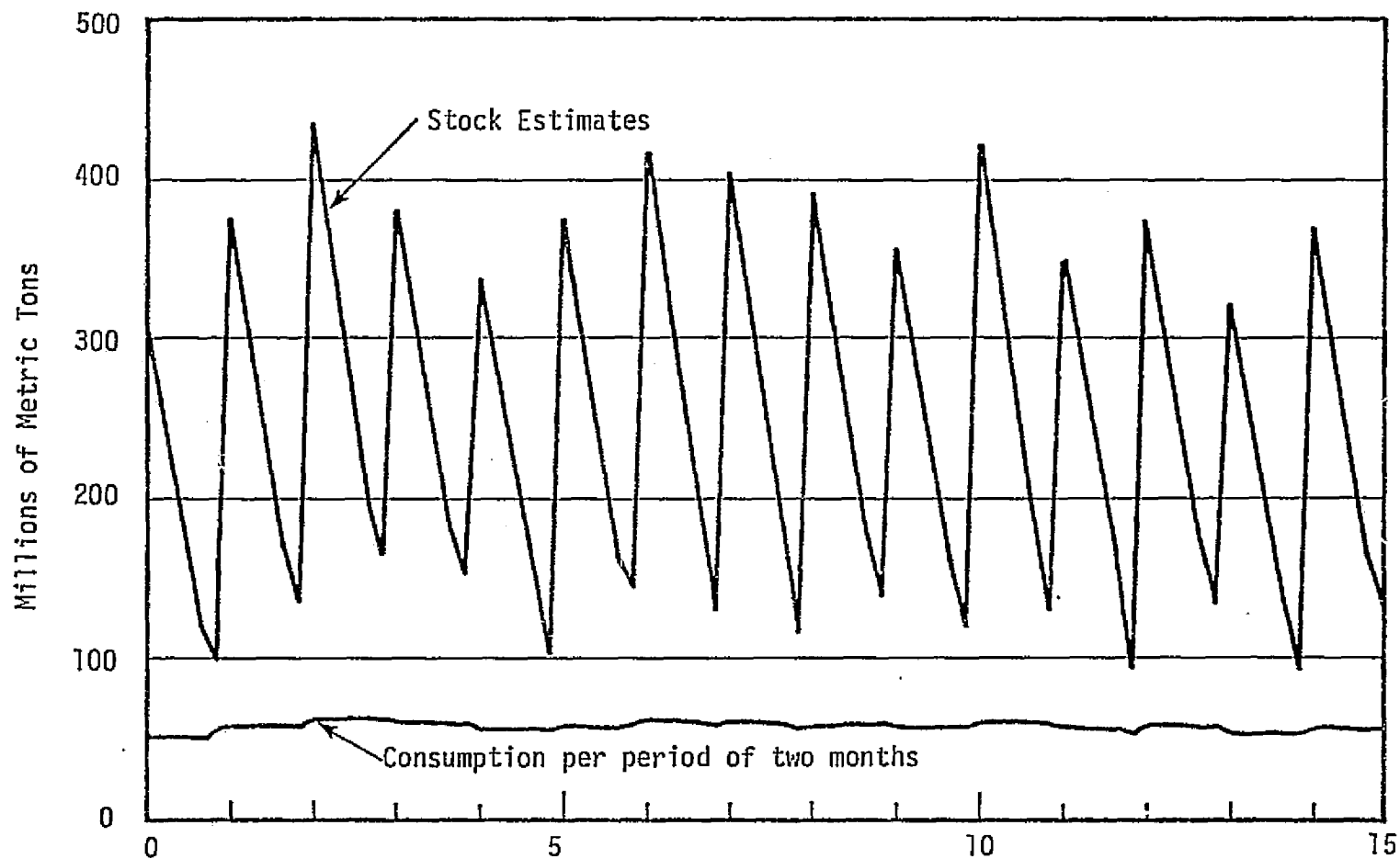


Figure 4.10 A Simulation Case--Rest of World Estimates of Stocks, Consumption. Information System Meeting LACIE Targets.

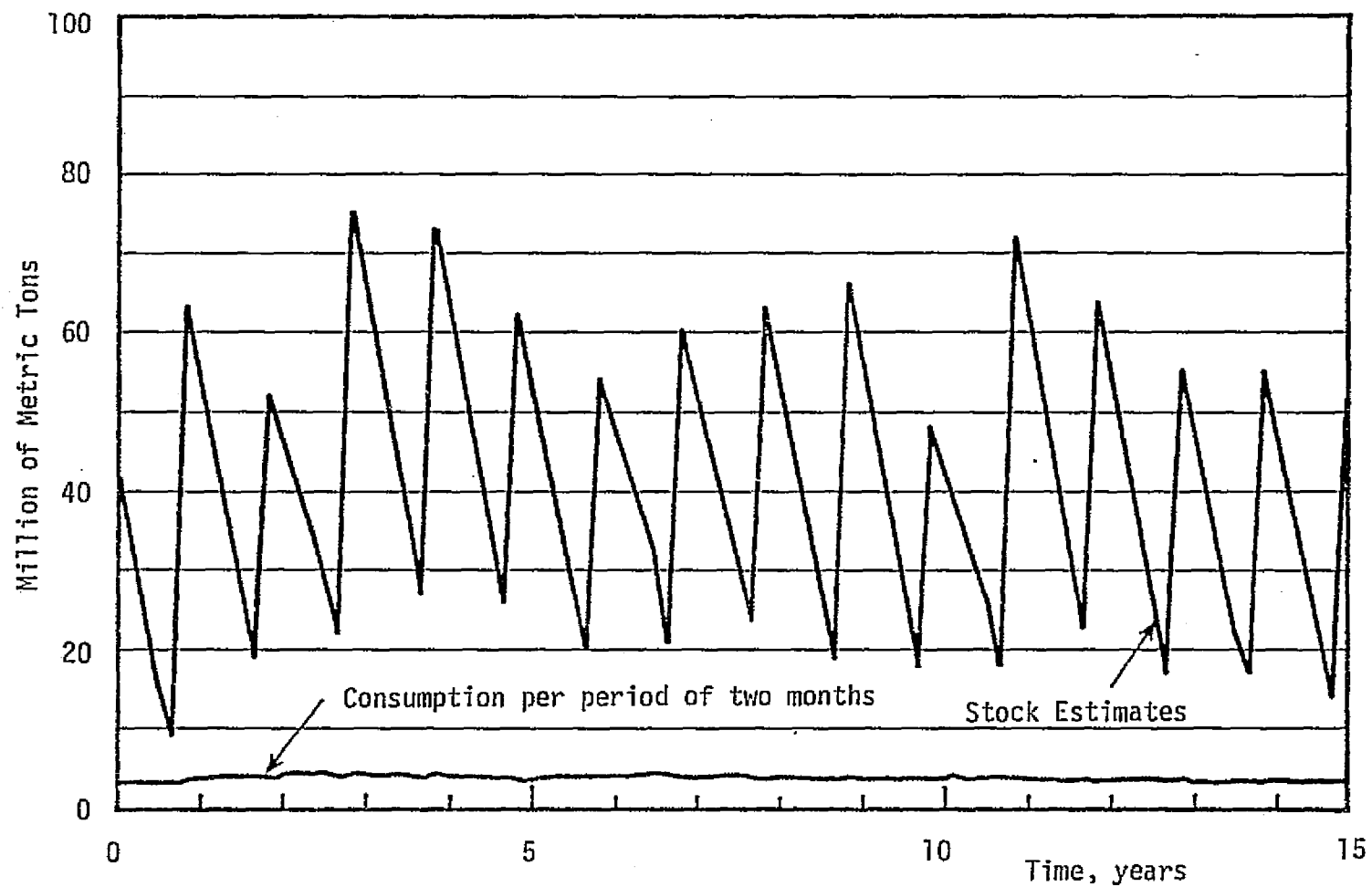


Figure 4.11 A Simulation Case--United States Estimates of Stocks, Consumption: Current Information System.

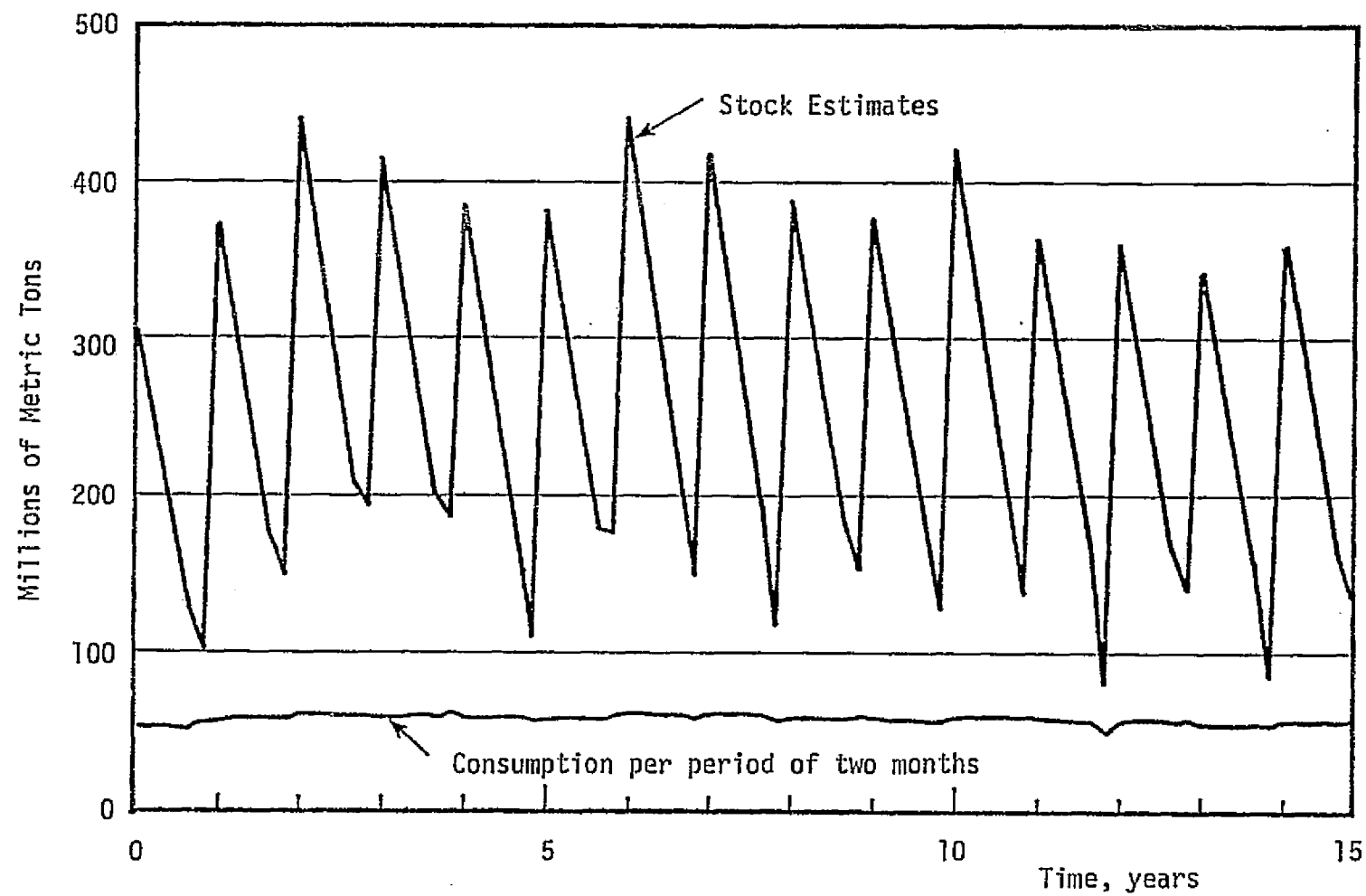


Figure 4.12 A Simulation Case--Rest of World Estimates of Stocks, Consumption. Current Information System.

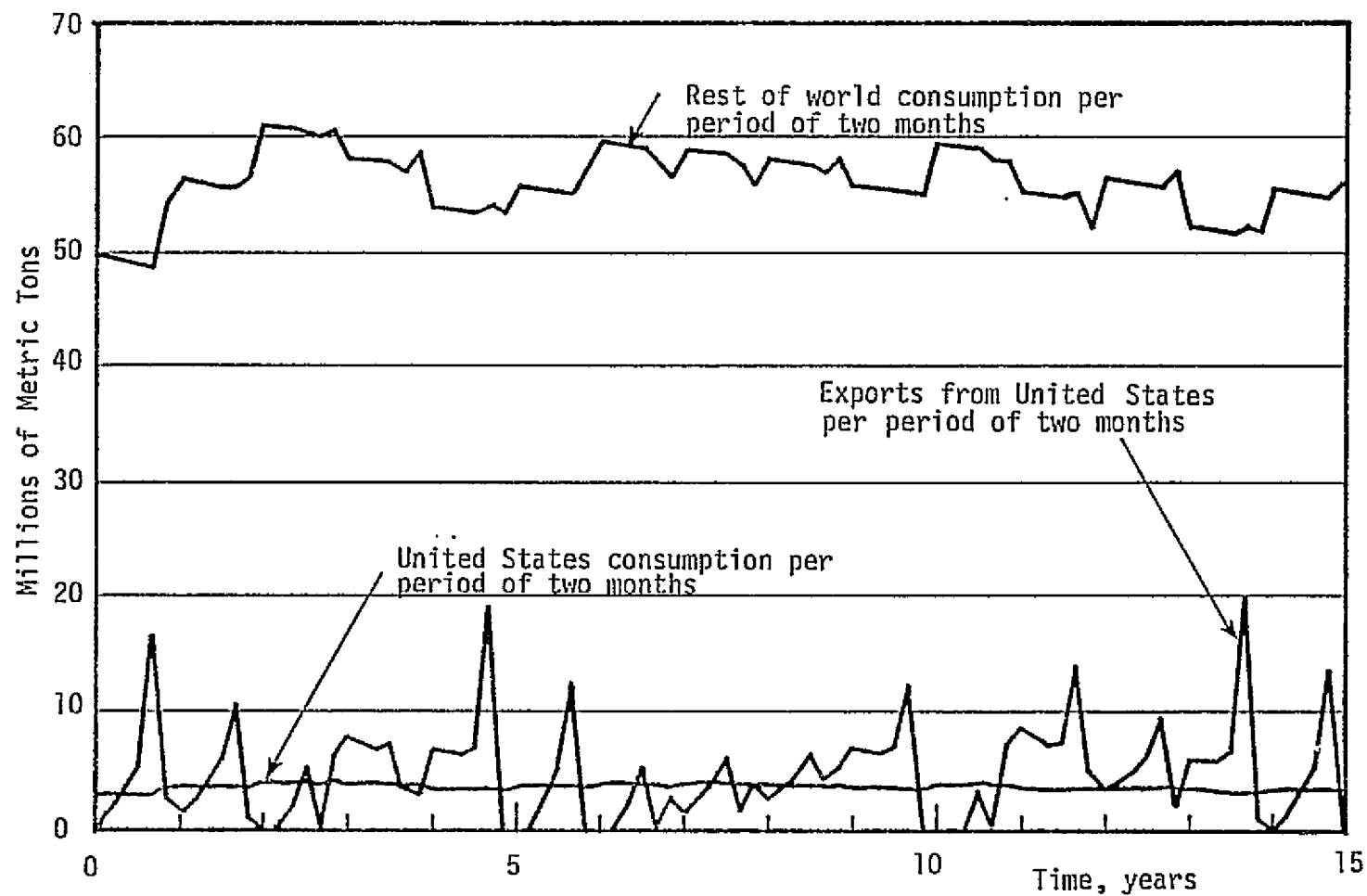


Figure 4.13 A Simulation Case-- Consumption and Exports.
Information System Meeting LACIE Targets.

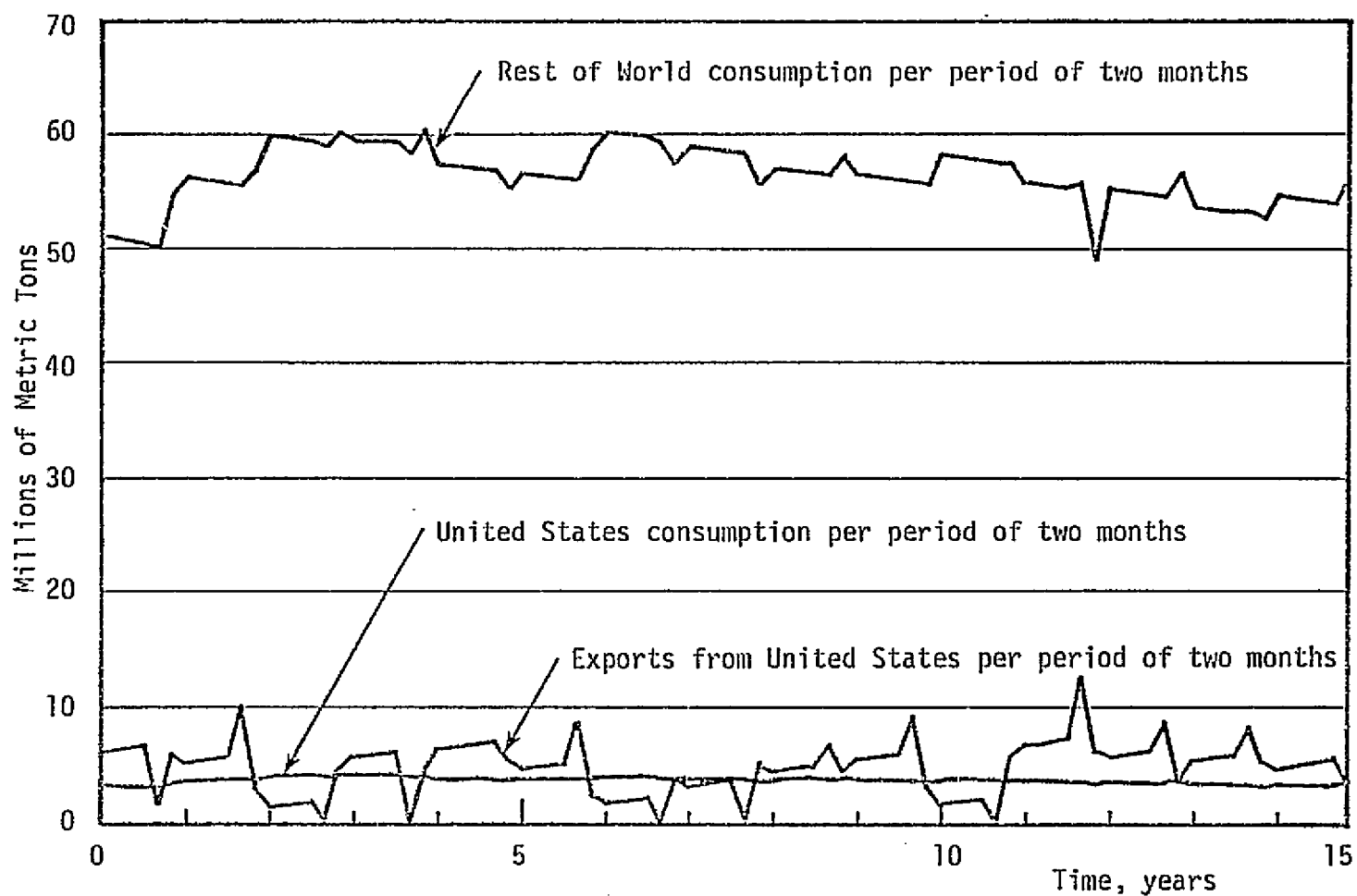


Figure 4.14 A Simulation Case--Consumption and Exports. Current Information System.

system, the average fifth period exports are lower than in any other period with the "LACIE" system, however, exports tend to concentrate in this fifth period. This is shown more clearly in Table 4.5, which gives the average exports and average United States prices by period. These averages are over the entire set of ten 30-year simulations.

Table 4.5 Mean Exports and Prices by Period						
Period Beginning	Current System			LACIE Targets		
	Mean Exports, millions of metric tons	Std. Dev. of Exports, millions of metric tons	Mean Price, 1975 \$/ metric ton	Mean Exports, millions of metric tons	Std. Dev. of Exports, millions of metric tons	Mean Price 1975 \$/ metric ton
August 1	4.95	1.75	154	3.18	3.08	157
October 1	5.12	1.77	155	3.60	2.60	158
December 1	5.29	1.79	157	4.48	1.88	159
February 1	5.47	1.80	158	6.08	0.97	160
April 1	4.35	3.90	187	9.69	6.46	164
June 1	4.76	1.29	152	2.73	2.67	155

5. CONCLUSIONS

The work described in this report was done primarily in order to validate the extensions and refinements of modeling methodology required for distribution benefit studies of worldwide information on crops other than wheat. The application to wheat in Chapter 4 was chosen because previous ECON studies have provided numerical estimates for comparison with the current results.

The numerical results are consistent with the study of total LANDSAT benefits [5]. Our numerical estimates for wheat could be made more precise by a detailed analysis of the development and release of information throughout the crop year, both for the current information system and for the improved system under study. Such a detailed analysis is part of the plan for the application of the model to other crops in the coming months.

Concerning the modeling methodology, we have found that it is important that the decision rule assumed represent the ability of free market agents to use all available information and plan with an unlimited horizon. Any lesser assumption we experimented with led to significantly different results. Thus, for example, iterative solution method described in Section 3.6 is an essential part of our methodology. We

found that the alternative of setting boundary conditions by heuristic criteria was completely unsatisfactory.

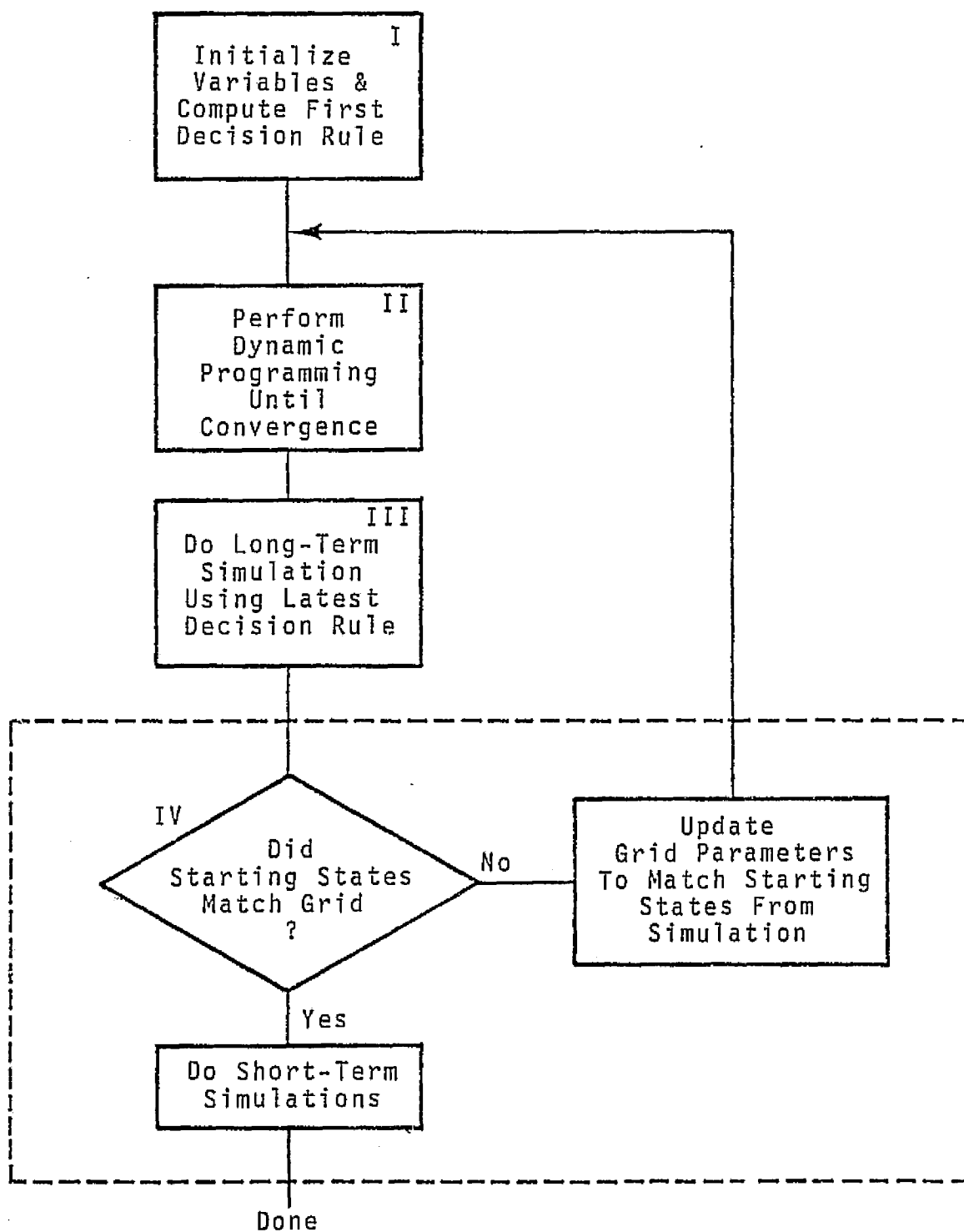
The use of parametric representation of the value functions with second degree polynomials was the most satisfactory approach of those we experimented with. The value functions can also be stored at selected points of the state space, but then the maximizations must be done by 3-dimensional search techniques, which are extremely time consuming. Besides the calculation expense, such a technique has the difficulty that interpolation errors can easily obscure the value of information terms, which are second order effects.

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"APPENDIX A & B NOT INCLUDED"

For each information system, perform this procedure:



The Algorithm

For each system, current and LACIE, perform this procedure:

- Block I Initialize all program variables. The first period value function and approximate grid parameters are calculated based on a simple heuristic decision rule. The remainder of the algorithm improves these estimates of the value functions and period-by-period state distributions to find the optimal decision rule.
- Block II Using the given grid parameters, perform a dynamic programming routine until convergence is achieved. This procedure finds the optimal decision rules based on a particular set of grids.
- Block III Using the decision rules found in Block II, perform a long-term simulation. Collect various statistics, including the period by period means and variances of the starting state distributions.

Block IV If the simulated states do not match the grid parameters, then update the grid parameters with the means and variances of the state distributions and go to Block II.

If the grid and states do agree, then the optimal long-term decision rule has been found. Using them, perform ten short-term simulations and average them.

Note that wherever n is used as a time-dimensional subscript, it is taken as $\text{mod}_m(n-1) + 1$. That is, if $n = m$, then $Q_{n+1} \equiv Q_1$.

The random number generator was reset for each long-term simulation, and both systems ("current" and "LACIE") were analyzed using the same corresponding random numbers. Thus the comparison is for the same sequences of grid point selections in Block II, Step 2c.

Procedure Block I

INITIALIZATION

Notes: The actual computed values are supplied in parentheses. All quantities are in millions of metric tons, and prices are in 1975 dollars per ton. Unit 1 is the United States (exporting unit), and 2, the rest of the world (importing unit).

Step 1 Input.

1. $e_v(v = 1, 2)$ = annual price elasticities for unit v .
($e_1 = -0.5$, $e_2 = -0.3$).
2. r = annual discount rate. (0.062)
3. m = number of periods per year. 1 year/ m must be the average interval between order and delivery of exports. (6)
4. τ , τ_1 = coefficients of transportation cost equation. $\text{Cost} = \tau e + \tau_1 e^2$ where e is annual quantity of exports. (8, 0.05517)
5. $\pi_{nv}(n = 1, 2, \dots, m; v = 1, 2)$ = average quantity of wheat harvested during period n in unit v .

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 50 & 0 \\ 0 & 300 \end{pmatrix}$$

Note the one period difference in the harvest periods for the two units.

6. $\sigma_{nv}^2(F)(n = 1, 2, \dots, m; v = 1, 2) =$ variance of forecast to be available at time $n + 1$, conditional on the currently available forecast for unit v . For $\sigma_{5,1}^2(F)$ and $\sigma_{6,2}^2(F)$, the variance is the unconditional variance of the first forecast of annual production. These are the only input items that distinguish the two systems.

Current	LACIE
$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 9 & 0 \\ 27 & 1000 \\ 0 & 1000 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 9 & 0 \\ 27 & 333 \\ 0 & 1667 \end{pmatrix}$

Step 2 Annual Demand Function Parameters.

1. $\alpha = \frac{4.125}{e_1} = \frac{1}{2}$ slope of exporting unit's demand function, expressing price as a function of annual consumption. (-8.25)

$$2. \quad \beta = 165(1 - \frac{1}{e_1}) = \text{Intercept of exporting unit's demand function. (495)}$$

$$3. \quad \gamma = \frac{0.262}{e_2} = \frac{1}{2} \text{ slope of importing unit's demand function, expressing price as a function of annual consumption. (-0.87333)}$$

$$4. \quad \delta = 173(1 - \frac{1}{e_2}) = \text{Intercept of importing unit's demand function. (749.666)}$$

Step 3 Data adjustment for lengths of periods.

$$1. \quad \rho = (\frac{1}{1+r})^{\frac{1}{m}} = \text{discount factor for a single period. (0.9900244)}$$

$$\begin{aligned} 2. \quad \alpha &\leftarrow m\alpha & (-49.5) \\ \gamma &\leftarrow m\gamma & (-5.24) \\ \tau_1 &\leftarrow m\tau_1 & (0.33102) \end{aligned}$$

Step 4 Definition of constant matrices.

$$A = \begin{pmatrix} \alpha & 0 & \alpha \\ 0 & 0 & 0 \\ \alpha & 0 & 0 \end{pmatrix} \quad B = \begin{pmatrix} \beta \\ 0 \\ \beta \end{pmatrix}$$

$$E = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \gamma & 0 \\ 0 & 0 & \tau_1 \end{pmatrix} \quad F = \begin{pmatrix} \beta \\ \delta \\ -\tau \end{pmatrix}$$

$$M = \begin{pmatrix} -1 & 0 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

Step 5 Approximate Grid Parameters.

$$1. \quad \epsilon = \frac{2\alpha \left(\sum_{n=1}^m \Pi_{n,1} \right) - 2\alpha \left(\sum_{n=1}^m \Pi_{n,2} \right) + m(\tau + \beta - \delta)}{2(\alpha + \gamma) - \tau_1}$$

= estimate of average annual exports. (29.924)

2. $\mu_{nv}(G)(n = 1, 2, \dots, m; v = 1, 2)$ = estimate of the mean states at time n for unit v . The grid will be centered on these means. Initially, $\mu(G)$ is calculated according to even consumption and export decisions and a stock carry-over level equal to

$$\left(\sqrt{\sigma_{4,1}^2(F)}, \sqrt{\sigma_{5,2}^2(F)} \right).$$

$\mu(G)$ is updated in Block IV until it agrees with the simulation results gathered in Block III.

(Initially, $\mu(G)$ =

$$\begin{pmatrix} 44.667 & 323.236 \\ 36.333 & 273.236 \\ 28 & 223.236 \\ 19.667 & 173.236 \\ 11.333 & 123.236 \\ 53 & 73.236 \end{pmatrix} .)$$

3. $\sigma_{nv}^2(G) (n = 1, 2, \dots, m; v = 1, 2)$ = estimate of the variance of the states at time n for unit v . The grid spread will be determined by these parameters. Initially,

$$\sigma^2(G) = \mu(G) \times \begin{pmatrix} 0.5 \\ 0.4 \\ 0.3 \\ 0.2 \\ 0.2 \\ 0.2 \end{pmatrix}.$$

4. $S_v(F) (v = 1, 2)$ = Current state at time $n = 1$. $S(F)$ is calculated similarly to the presumptive state, $\mu_1(G)$, with a zero carry-over level assumption. All simulations start from $S = S(F)$.

Step 6 Initialize Value Function

(only period 1 is needed).

$$Q_1 = \begin{pmatrix} \frac{\alpha}{25(\frac{1}{\rho}-1)} & 0 \\ 0 & \frac{\gamma}{36(\frac{1}{\rho}-1)} \end{pmatrix}$$

$$P_1 = \begin{pmatrix} \frac{2\varepsilon\gamma + m\delta}{6m(\frac{1}{\rho}-1)} \\ \frac{-2\varepsilon\gamma + m\delta}{5m(\frac{1}{\rho}-1)} \end{pmatrix}.$$

$$R_1 = \frac{\frac{\epsilon^2}{m^2} (\alpha + \gamma - \tau_1) + \frac{\epsilon}{m} (-\beta + \delta - \tau)}{(\frac{1}{p} - 1)}$$

Step 7 EXIT BLOCK.

Procedure Block II

DYNAMIC PROGRAMMING

Step 1 $n \leftarrow m$

Step 2 Select 25 states (grid points)

$$S_{ij} \quad (i = 1, 2, \dots, 5; j = 1, 2, \dots, 5) =$$

$$\mu_n(G) + \sqrt{2} \times \begin{pmatrix} i-3 \\ j-3 \end{pmatrix} \times \sqrt{\sigma_n^2(G)}$$

Step 3 For each S_{ij} in Step 2, find the 1-stage optimal decision vector $\hat{Y}$ by maximizing

$$W = Y^*ZY + Y^*G$$

subject to the constraints

$$Y \geq 0,$$

$$Y_1 + Y_3 \leq \max(0, S_1 - \sqrt{\sigma_{n,1}^2(F)})$$

$$Y_2 \leq \max(0, S_2 - \sqrt{\sigma_{n,1}^2(F)})$$

where $Z = \rho M^* Q_{n+1} M + E$ and

$$G = \rho M^* (2Q_{n+1}(S_{ij} + \Pi_n) + P) + F.$$

Store the maximands (S's) and the maxima (W's) for use in Step 4.

Step 4 Do the least squares fit of the quadratic form

$$SQ_n S + SP_n + R$$

to the maxima calculated in Step 3. (When all the maximizers in Step 3 are unconstrained, the LSF has zero residual.)

This step generates the new period n value function parameters.

Step 5 $n \leftarrow n - 1$; if $n \geq 1$ then go to Step 2.

Step 6 Check for convergence: If Q_n, P_n, R_n ($n = 1, 2, \dots, m$) have changed significantly since the last iteration, go to Step 1.

Step 7 EXIT BLOCK.

Note: This procedure usually converged in three iterations and at most in seven.

Procedure Block III

LONG-TERM SIMULATION

Using the value function parameters Q , P , and R generated by the dynamic programming in Block II to determine the decision rule, simulate 120 years of decision making and summarize the results, ignoring the contributions of the first 20 years.

Step 1 Initialize the state $S \leftarrow S(F)$.

Step 2 Perform this procedure once for each year of the simulation (120 times), recording decisions, prices, and starting states for each period simulated past the 20th year. (This allows the state to reach the long term equilibrium.)

Step 2a. $n \leftarrow 1$.

Step 2b. Find the optimal decision vector $\hat{Y}_n$ based on the starting state S_n .
(See Block II, Step 3.)

Step 2c. $S_{n+1} \leftarrow S_n + M\hat{Y}_n + \Pi_n + \Phi_n$

Where Φ_n is a vector selected at random from a rectangular distribution of 25 points with means in both

dimensions of zero and variances equal to the forecast variances, $\sigma_n^2(F)$.

Step 2d. $n \leftarrow n + 1$; if $n \leq m$ then go to Step 2b.

Step 3 Calculate, print, and optionally plot the following statistics:

Means and variances of

1. Unit 1's consumption decisions by periods.
2. Unit 2's consumption decisions by periods.
3. Export decisions by periods.
4. Export decisions by year.
5. Starting states for each period. These are used in Block V to set the new grid parameters, $\mu(G)$ and $\sigma^2(G)$.
6. Prices by period and unit.
7. All prices by unit.
8. Discounted annualized values of consumption by unit.

Step 4 EXIT BLOCK.

Procedure Block IV

Step 1 Check convergence of the grid fitting procedure.

If the grid parameters, $\mu_n(G)$ and $\sigma_n^2(G)$ ($n = 1, 2, \dots, m$), differ significantly from the means and variances of the starting states found in Block III, Step 3, line 5 (the long-term simulation), then replace $\mu_n(G)$ and $\sigma_n^2(G)$ with the simulated values and go to Block II.

If the grid parameters do match, however, then the best long-term decision rule has been found.

Step 2 Perform ten simulations of 30 years each, following the same procedure as described in Block III, collecting the simulation data for all 30 years. Print the averages of the results from the ten short-term simulations.

Step 3 END.

APPENDIX B

VALIDITY OF BENEFIT MEASUREMENT BY CONSUMER SURPLUS

The market demand curves used to quantify economic value in this study are essential analytic tools.

First, a market demand curve is in effect a sales possibility frontier and as such constrains the pricing strategies that could, in principle, be followed by the producer of a good or service. Second, the demand curve furnishes probably the most objective - although by no means the only - measure of the aggregate value society as a whole attaches to a given output rate q of the commodity being produced. Third, in like manner, the demand curve furnishes the most objective measure of value of the added incremental or marginal unit of output at any given output rate q . [If q is measured in discrete units, this so-called incremental or marginal value at output rate q is the maximum anyone in society would be willing to offer for the $(q + 1)$ st unit.]

The proposition that the market demand curve for a product furnishes a meaningful measure of that product's value to society rests on a number of normative precepts that are not universally shared - certainly not by totalitarian societies, and frequently not even by proponents of democratic government. Since this point is so often overlooked in modern economic analysis, it may be well to identify these normative precepts at the outset.

The derivation of an individual's demand curve and its relationship to value are illustrated with the aid of Figure B-1 and B-2. Figure B-1 illustrates how the total value user A derives from alternative use rates (q_A) varies with changes in the use rate. The curve embodies the hypothesis that the value added by successive increments in the use rate diminishes, at least after a point. This assumption is commonly made in economic analysis and is generally realistic. If the user in question is a consumer and the product a consumer good, diminishing marginal value seems intuitively obvious. If the product is an intermediary good, its marginal value may remain constant over a substantial range, although ultimately its use may be subject to diminution in marginal value. These increments in value are plotted in Figure B-2. If the total value line in Figure B-1 were a smooth curve, then that in Figure B-2 would represent the first derivative of the total value curve with respect to q_A .

The market demand curve of a product - as that analytic concept is used in economic analysis - is simply the horizontal addition of the demand curves characterizing the valuation individual, potential users of the product place upon alternative use-rates of the product per period. Figure B-3 depicts this derivation for a "market" composed of only two potential users, A and B.

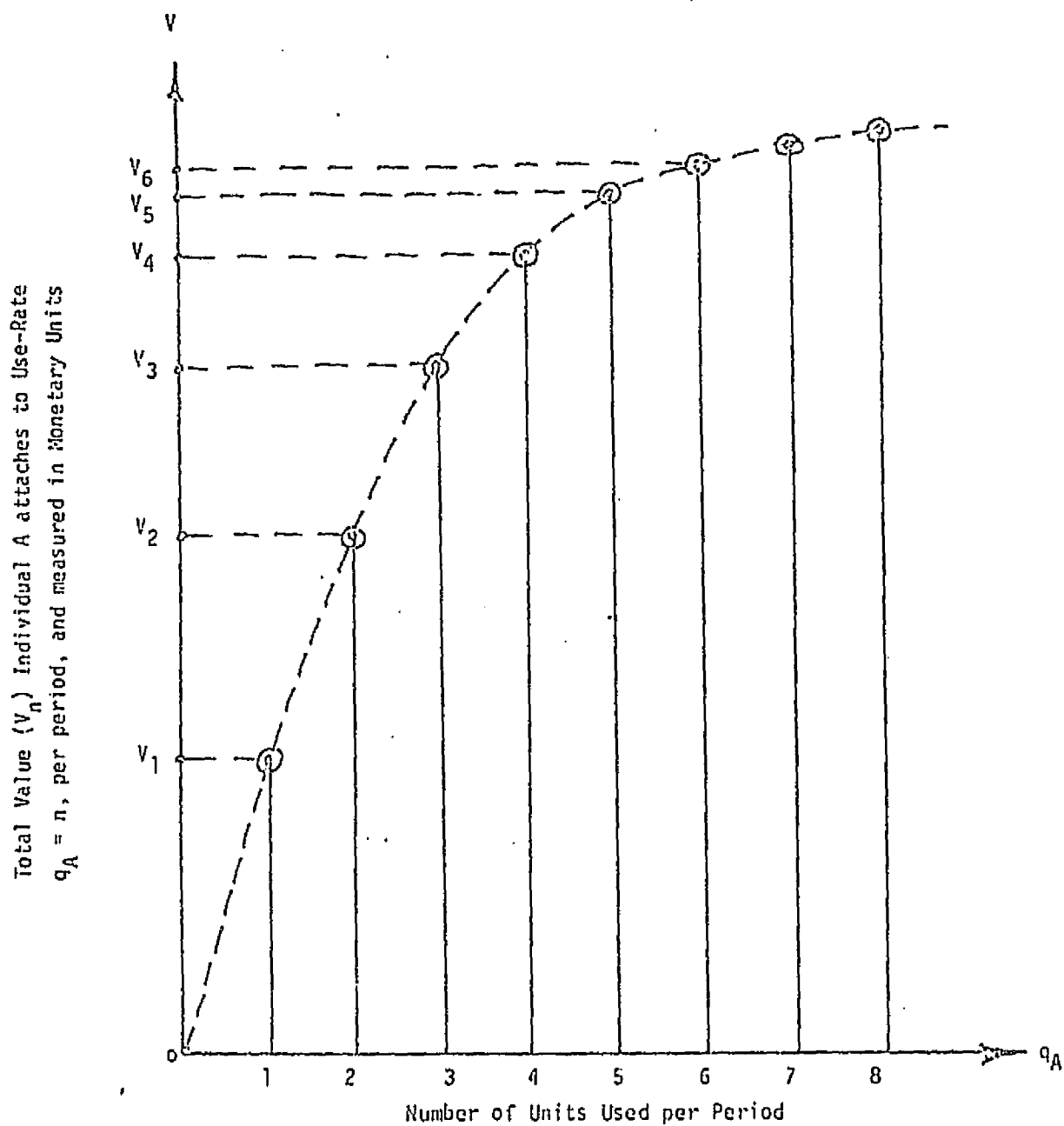


Figure B-1 Total Value Attached to Alternative Use Rates by Individual A.

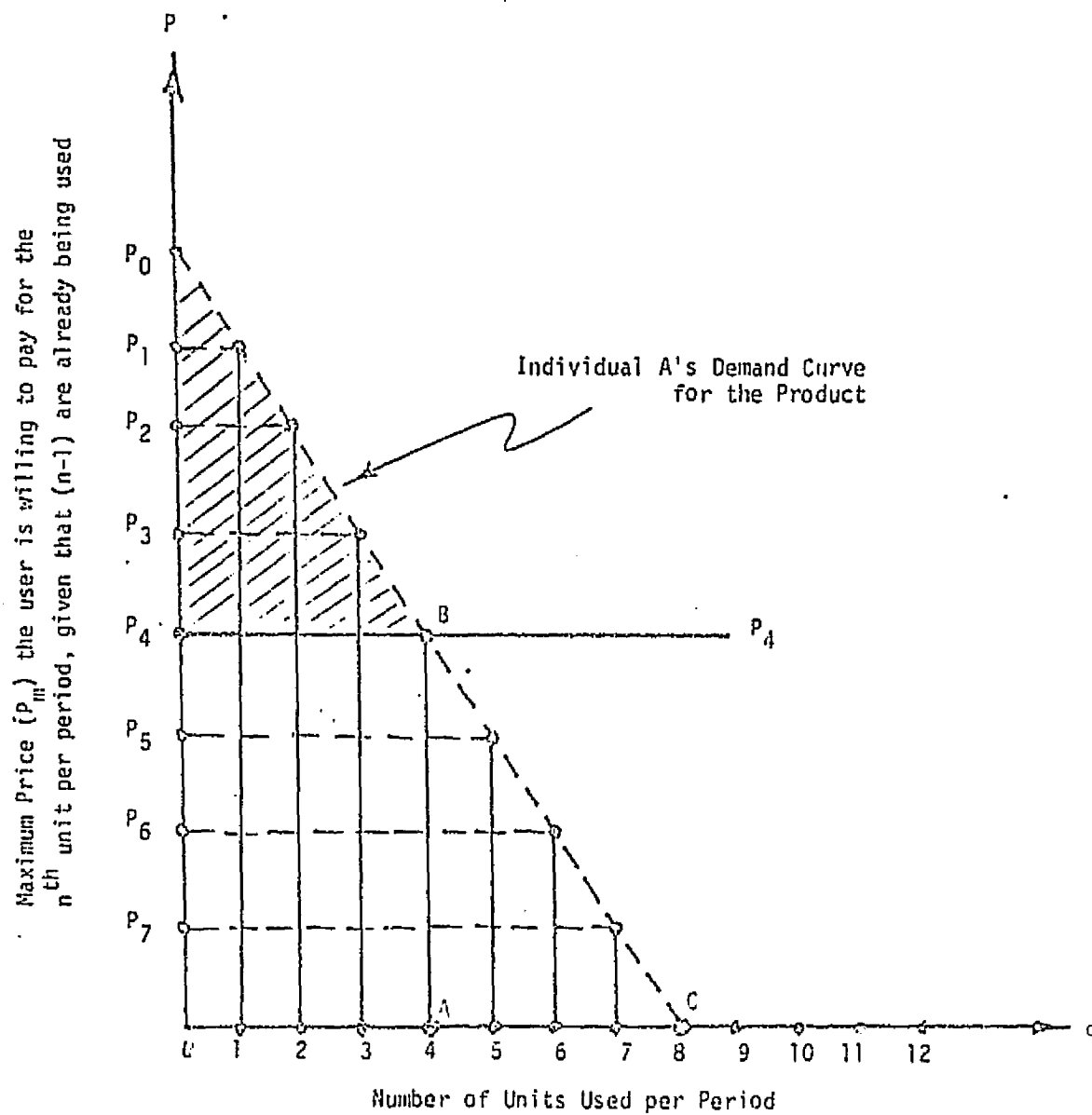


Figure B-2 Marginal Value attached to n^{th} Unit of Use per Period by Individual A.

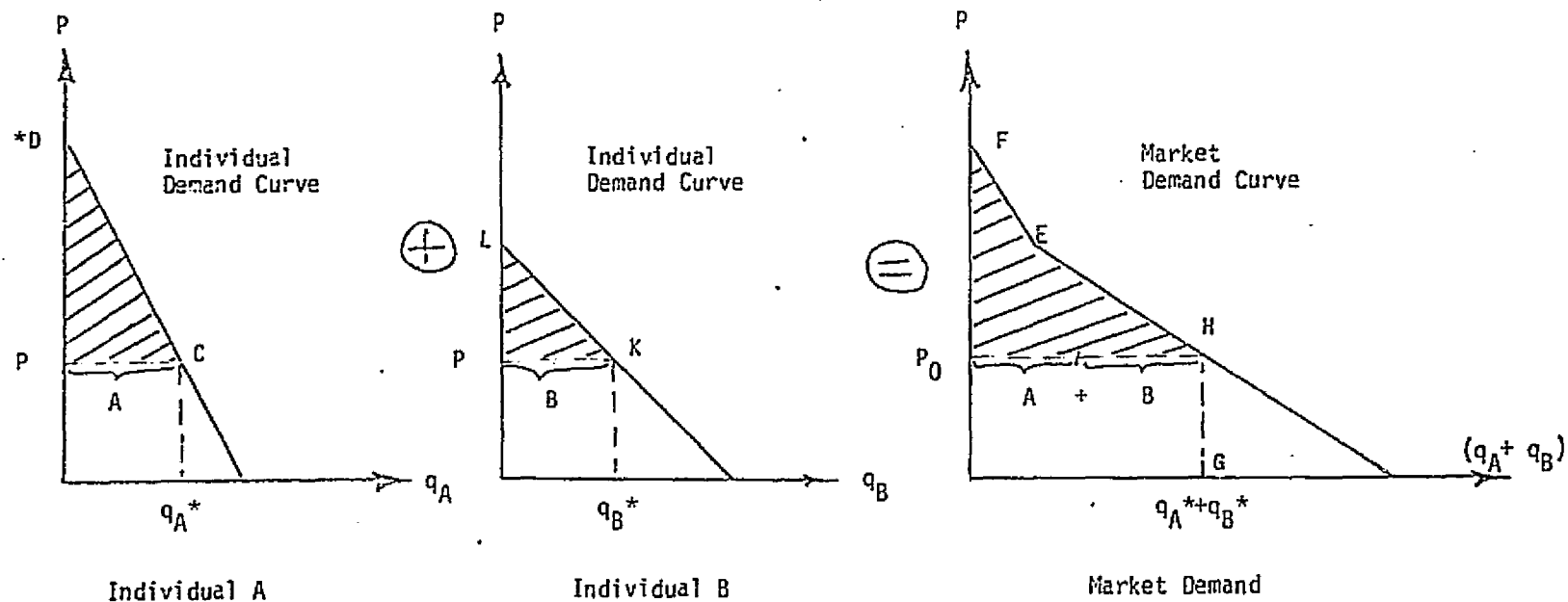


Figure B-3 Derivation of the Market Demand Curve

It is easily shown that the marginal value curve from p_0 to C depicted in Figure B-2 also represents user A's demand curve for the product in question. An individual's demand curve for a product depicts the use rates, q , he would choose at given, alternative prices, p . Now suppose user A were confronted with a price P_4 and acted with the simple, total value maximizing rule to purchase additional units per period as long as the value to him of the marginal unit exceeded its cost (that is, its price). Quite obviously, the individual would thus be driven towards use rate $q_A = 4$ at P_4 , towards $q_A = 5$ at P_5 , to $q_A = 3$ at P_3 , and so on. Obviously, then, an individual user's marginal value curve for a product is identical with his demand curve for that product.

Consider now a use rate $q_A = 4$. In Figure B-1 the total value (V_4) user A associates with that use rate can be read off the vertical axis. In Figure B-2 that value would be the sum $P_1 + P_2 + P_3 + P_4$. If the total and marginal value curves were continuous lines (that is, if q_A were continuously variable) then the total value V_4 would be given in Figure B-2 by the area $O A B P_0$ under the marginal value curve. By assumption, however, the user can obtain $q_A = 4$ units per period in exchange for paying price P_4 per unit of the product. This total cost $P_4 \cdot 4$ is given by area $O A B P_4$ in Figure B-2. Clearly, user A pays less for the product than the total value (area $O A B P_0$) he attaches to it at $q_A = 4$. The difference between total value and total

cost to individual A - the shaded triangle $P_4B P_0$ in Figure B-2 - is known as "consumers' surplus" in the economics literature. As will be noted further on, the concept of consumer's surplus plays a central role in discussions on the resource allocative efficiency in a market economy. Resources in an economy can be said to have been allocated efficiently when their use maximizes consumer's surplus in the economy.

The alert reader will have noted the subtle switch from "consumer's surplus" (singular) to "consumers' surplus" (plural) in the preceding paragraph. The individual consumer's surplus reflects his own preferences, his own choice process, and, quite obviously, also the budget that constrains his choices: A potential consumer of a product totally without resources clearly could not afford to pay any positive price for any unit of the product - his demand curve would collapse into the origin of Figure B-2, even if his need for the product were acute. Loosely speaking, the concept of consumers' surplus for society as a whole can be defined as the total social value society attaches to given aggregate use rates q minus the total social cost of producing that output rate. The reader may have asked himself how the aggregation from the individual's value to social value is performed conceptually and in practice.

The answer to this question must be that economists have not so far been able to perform this aggregation

satisfactorily, even at the conceptual level. One must add the prognosis that the problem is unlikely ever to be solved. For practical purposes, economists commonly measure the social value of a product at given output rates by the corresponding area under the market demand curve. Thus, in Figure B-3 the total social value (per period) imputed to output rate ($q^*_A + q^*_B$) would be the area $O G H E F$. If the output in question were produced at constant social costs per unit equal to P_0 , then area $O G H P_0$ would be the corresponding total social cost of output rate ($q^*_A + q^*_B$) and area $P_0 H E F$ would be the consumer's surplus for this market. This measure of aggregate (social) consumers' surplus is seen to be simply the arithmetic sum of the individual consumer's surpluses, namely, area $P_0 C D$ for consumer A and area $P_0 K L$ for consumer B.

Many social philosophers - some economists among them - object to the preceding definition of social value because measures based on these definition are rooted in part in a specific distribution of purchasing power (budgets, wealth and income) among individuals in society. These measures of social value are therefore only as defensible, ethically, as is the particular underlying distribution of purchasing power. A change in that distribution will normally alter also the total social value one imputes on this approach to a given output rate q . It is a circumstance that destroys the absolute applicabilities of the economist's approach to this measurement problem. Figure B-4 illustrates this subtle point graphically.

In Figure B-4, we continue to assume that the "market" in question is composed of two individuals, A and B. The solid demand curves are drawn for a given distribution of income and wealth between A and B. Suppose now that a lump sum of purchasing power is taxed away from A and transferred to B. This transfer of purchasing power will shift the demand curves to the positions indicated by the broken lines. At any given price for the product, the now relatively impoverished user A would demand fewer units of the product per period. Given other needs pressing on his budget, he would attach a somewhat lower monetary value to an additional unit of this particular product at given use-rates q_A . Individual B, on the other hand, would now be relatively better off. At any given price, he would demand more of the product per period or, equivalently, at any given output rate q_B he could now afford to attach a higher monetary valuation to the marginal unit of the product. The magnitudes of these demand curve shifts would depend on these individuals' so-called "income elasticities" of their demand for the product. This elasticity is a measure of the response to the demand for the product to changes in income. Precisely defined, it is measured as the percentage change in quantity demanded over the percentage change in income, other things being equal. The point to note is that as a result of the distribution of income and wealth the market demand curve for the product may shift either up or down at given aggregate use rates q . In the case illustrated

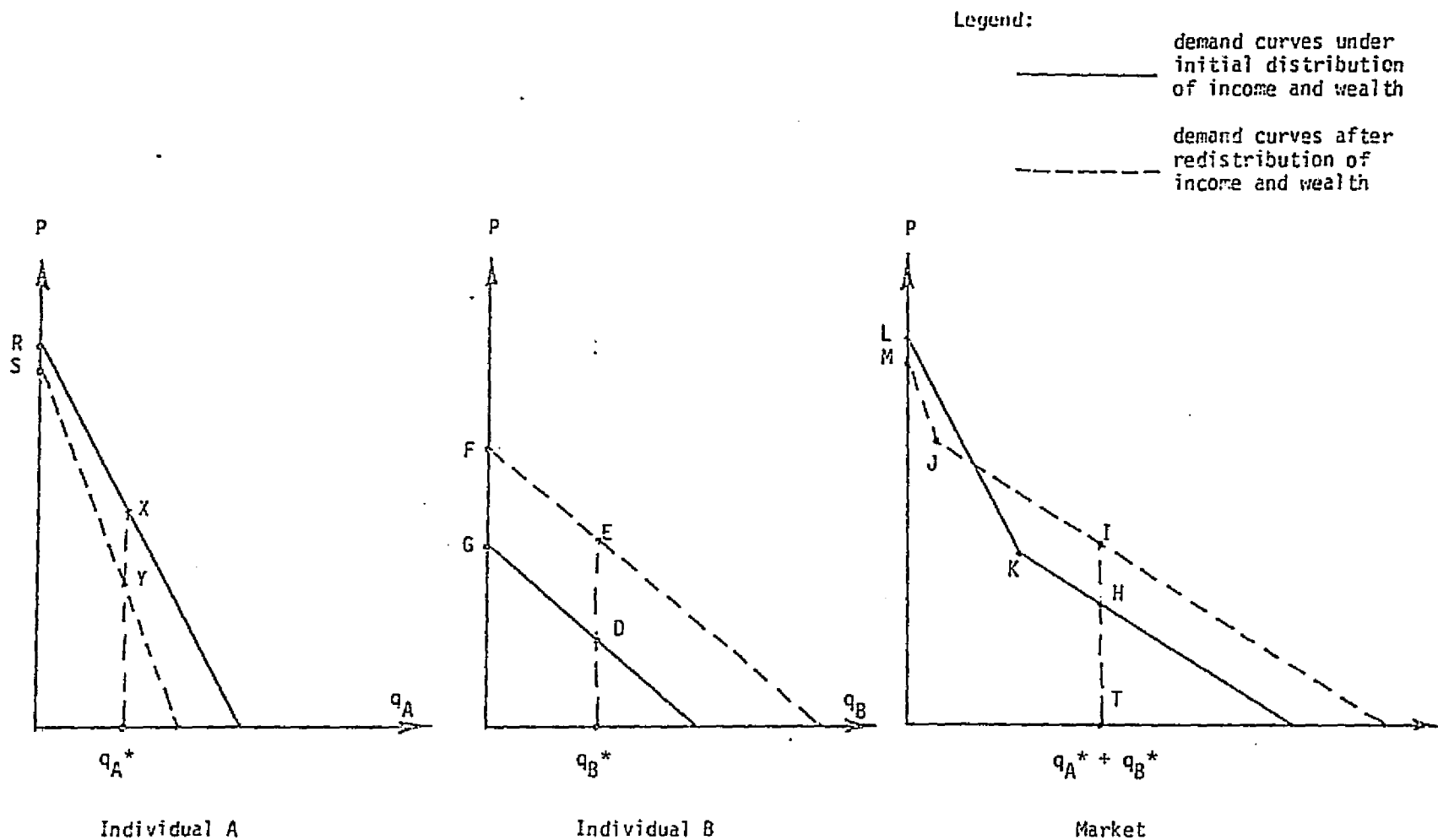


Figure B-4 Illustration of the Effect of a Change in the Distribution of Income and Wealth on the Social Value of a Product

with Figure B-4 the redistribution had the effect of raising the imputed social value of the product at output rate $q^*_A + q^*_B$, as can be seen by comparing area O T H K L with area O T I J M. It might have reduced the imputed value if individual A had a strong preference for the product but viewed it as a luxury item (in which case the income elasticity of his demand for the product would be high and reductions in his income would result in a substantial downward shift of his demand curve) and if individual B's demand for the product were relatively income inelastic.

This observation on "income elasticities" and their effect on a distribution of value to society is far from being of purely theoretical interest in the context of our study. Wheat (and also information on wheat) has been shown to be an "inferior" good in demand studies within any one country and between countries. The term "inferior" in this case has no moral stigma attached to it, but simply means that the higher an individual's income, or a country's income, the lower the relative demand for wheat. The value attached to wheat, and to information on wheat, will be relatively higher to low income areas and groups than to higher income groups and countries. Since the evaluation in this study is primarily carried out to determine benefits to the United States -- one may expect a relative underestimation of the true value of improved wheat crop information to the rest of the world.

A further point to be drawn from this illustration is that the allegedly absolute measures of values economists derive from market demand curves do, in fact, contain a relative element, namely, the purely normative proposition that the distribution of income and wealth underlying these demand curves is given and ethically defensible. It is well known that many commentators in the social sciences, in government, and in the media strongly disagree with that proposition. Logical consistency alone would lead these commentators to question the economist's approach to valuation, as a good many of them do.

Some critics go so far as to question the economist's methodology on grounds quite distinct from an objection to the prevailing distribution of income and wealth. Even if that distribution were satisfactory, they argue, it would still be illegitimate to derive measures of the social value of a product from the corresponding market demand curves. This proposition is compatible with two rather distinct social philosophies, only one of which is compatible with political democracy.

According to the first of these philosophies, social values are properly derived from the valuations arrived at by individual members of society, but the total social value of a product need not be the arithmetic sum of individually imputed values (as was assumed in Figures B-3 and B-4 above). First, it is possible that the use of a product by one individual generates positive or negative spillover effects on other

members of society, so that the individual user's valuation of the product may under or overestimate its total social value. To the extent that the air pollution generated by smokers irritates non-smokers, for example, the market demand curve for tobacco products can be said to overstate the total social value of these products. Second, there is the possibility that some products cannot be competently evaluated by their ultimate users. Unless expert judgement is permitted to intervene, the values implicit in market demand curves may therefore deviate sharply from the valuations that would be imputed by competent experts. The market demand curves for health services or for pedagogic services, for example, are often said to reflect improper valuations of this sort. Indeed, a common complaint among the producers of professional services - be they physicians, educators, technicians or humanists - is that the ultimate beneficiaries of these professional services typically undervalue them, and that only the professional him or herself possesses the competence to impute the true social value of his or her own services. Clearly, that school of thought finds repulsive the notion that social values can always be reliably inferred from market demand curves - hence the reluctance among educators, for example, to draw parallels between their industry and the commercial sectors of the economy. Yet this set of objections to using market and individual - demand curves for determining the value to society do not seem to apply to the case of wheat and the other

food crops: externalities - benefits or costs - do not seem to exist, and wheat as well as other food crops are nearly evaluated - in general - by most consumers.

The school of thought referred to in the preceding paragraph is a distinct cousin of a philosophy according to which social values do not reside in the valuations arrived at by individual members of society in the first place. This statist philosophy views the entity called "society" as so much more than a mere sum of its individual members that the individual's preferences - and the market demand curves generated by individual preferences - become irrelevant to an evaluation of social choices. According to this school of thought, social values can be properly imputed only by the few visionaries who emerge as leaders of society - be it a community or a nation. Quite obviously, this social philosophy is at variance with democratic ideals and is best suited to totalitarian governmental structures. For purposes of the present analysis it can be dismissed as not relevant.

To sum up at this point: In advising decision makers on alternative courses of action, economists favor choices that contribute to the attainment of "efficiency in the use of society's resources." Efficiency in the use of resources is desired because it enables society to maximize the "total social value" of goods and services obtainable from a given resource base. Alternatively, it enables society to minimize the "total social cost" of producing a given array of goods and services.

To render these intuitively appealing statements operational, one obviously must have an agreed upon measure of value to society. For practical purposes, economists have used as a working definition of the value to society of a product at output/consumption rate q the area under the market demand curve up to rate q . The objective of this discussion has been to show that this measure of value is predicated upon some subtle assumptions about the propriety of imputing value to society from the valuations by individuals and about the appropriateness of the underlying income distribution.

Economists have usually proceeded on the assumption that the prevailing income distribution in a democracy is itself a product of social and political choice that need not be questioned. On that assumption, any measure of social value based on the prevailing income distribution can be thought of as ethically and socially acceptable, too. Much of the discussion in this investigation rests implicitly on this line of reasoning. In the absence of that assumption, there simply does not exist any alternative and nearly as objective a measure of social value. By basing this investigation on the concept of the market demand function, we follow general principles of economic analysis in free market economics which -- particularly in the case of wheat -- are quite applicable and acceptable in any economic system to guide government decisions.